

Research school on Aperiodicity and Hierarchical structures in tilings 18-22 Sep 2017 Lyon (France)

# TILINGS IN LYON, 2017



LIP, Lyon, France 18 — 22 Sept 2017



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This booklet can be found online at https://tilingslyon2017.sciencesconf.org











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#### LECTURES

Laurent Bartholdi (ENS, Paris and University of Göttingen):

#### Self-similar groups.

**Abstract:** Self-similar groups are groups defined by a "self-similar action": an action on a set, such that actions on small parts of the set mimick the action on the whole set. In the first part, I will describe the class of self-similar groups, its important examples, and connections to other classes of groups as well as square tiling problems.

In the second talk, I will concentrate on spectacular properties of some self-similar groups: infinite torsion groups, and groups of intermediate word growth. These examples were given by Grigorchuk in the 1980's, and are still the topic of active research. These self-similar groups are "contracting": this is the basis of most proofs, by induction, as well as the source of important algorithmic properties.

In the third talk, I will explain a deep and fruitful connection, due to Nekrashevych, between selfsimilar groups and dynamical systems: there is a bijection between contracting self-similar groups and expanding self-maps of topological spaces. This has led to answers of long-standing open questions in complex dynamics.

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Edmund Harriss (University of Arkansas, Fayetteville):

#### Local rules for aperiodic tilings.

**Abstract:** The discovery of aperiodic sets of tiles, that could tile the plane but not periodically opened up a world in which the local properties of fitting shapes together could enforce long range behaviour. In many of the early examples the behaviour seemed to almost be magic. It was possible to prove the set of tiles was aperiodic, but the proof did not give much insight into why. For hierarchical tilings some general constructions have emerged that allow the creation of local rules for any substitution tiling, though these methods are a little brute force, often giving sets with huge numbers of distinct tiles.

In the first lecture I will describe the history of aperiodic tile sets and prove that the Penrose tiles with matching rules are aperiodic. In the second lecture I will show the general structures on which we might consider local rules, and use these to show the methods that create local rules for and substitution tiling. Finally in the third lecture I will discuss a large family of tilings discovered by Goodman-Strauss that provide a useful space for detailed exploration between the magic and the brute force. I will conclude with some discussion of how these local rules can be applied in many ways from art to mathematics.

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Jarkko Kari (University of Turku):

## Wang tilings and Cellular Automata: aperiodicity and computability.

**Abstract:** Cellular automata are discrete dynamical systems based on local, synchronous and parallel updates of symbols written on an infinite array of cells. They are the simplest imaginable massively parallel computing systems that operate under the nature inspired constraints of locality of interactions and uniformity in time and space. They can also exhibit the physically relevant properties of time reversibility and conservation laws if the local update rule is chosen appropriately. Closely related notions in symbolic dynamics are tiling spaces: sets of infinite arrays of symbols defined by forbidding the appearance anywhere in the array of some local patterns. In comparison to cellular automata, the dynamic local update function has been replaced by a static local matching relation.

These lectures present classical results about cellular automata, discuss algorithmic questions concerning tiling spaces and relate these questions to decision problems about cellular automata, observing some fundamental differences between the one- and two-dimensional cases.

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Lorenzo Sadun (University of Texas at Austin):

## Fusion : A general framework for hierarchical tilings.

**Abstract:** Many important classes of tilings exhibit a hierarchical structure. In this mini-course, we'll review the simplest setting, substitution tilings, where the hierarchy looks the same at each level. We will then relax the rules, allowing new structures at different length scales, and see that much (but not all!) of the theory of substitutions carries over with minor modifications. This includes topological constructions, spectral theory, and ergodic theory. We then extend the theory to cover tilings of infinite local complexity, and develop new measures of complexity to understand those examples.

## **STUDENTS TALKS**

## Mauricio Allendes Cerda (University of Chile):

## Continuous eigenvalues for Meyer sets.

**Abstract:** Let  $D \subset \mathbb{R}^d$  a Delone set and  $(X_D, \mathbb{R}^d)$  its dynamic hull system. Following [1] when the abelian group [D - D] is a finitely generated with r generators, we say that the rank of D is r and we write rank(D) = r. In this talk we will give a dynamical proof of the following result that appears in [3].

**Theorem 1.** Let  $D \subset \mathbb{R}^d$  be a repetitive Meyer set with rank(D) = r. The system  $(X_D, \mathbb{R}^d)$  has  $r \geq d$  continuous eigenvalues.

In dimension d = 1, following some ideas in [2], we will give conditions that ensure that every eigenvalue is continuous.

**Theorem 2.** Let  $D \subset \mathbb{R}$  be a repetitive Meyer set such that  $(X_D, \mathbb{R})$  is linearly recurrent. Suppose that exists a sequence of Kakutani-Rokhlin partitions such that for any integer  $m \ge 1$  the heights  $h_1(m), \ldots, h_{c(m)}(m)$  are rationally independent. Then  $(X_D, \mathbb{R})$  has only continuous eigenvalues.

We will exhibit some examples of dynamical system where Theorem 2 applies. Also we work two examples to observe that the hypotheses of being Meyer and having rationally independent heights are necessary in Theorem 2.

This is a joint work with Daniel Coronel.

## **References:**

[1] J. C. Lagarias. *Geometric models for quasicrystals I. Delone sets of finite type*. Discrete Comput. Geom. 21 (1999), no. 2, 161-191. MR 1668082.

[2] Cortez, Maria Isabel; Durand, Fabien; Host, Bernard; Maass, Alejandro. *Continuous and measurable eigenfunctions of linearly recurrent dynamical Cantor systems*. J. London Math. Soc. (2) 67 (2003), no. 3, 790-804.

[3] Johannes Kellendonk; Lorenzo Sadun. *Meyer sets, topological eigenvalues, and Cantor fiber bundles.* J. Lond. Math. Soc. (2) 89 (2014), no. 1, 114-130.

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Ilya Galanov (Université Paris 13, LIPN):

## On self-assembly of aperiodic tilings.

**Abstract:** Aperiodic tilings serve as a mathematical model for quasicrystals, such crystals that do not have any translational symmetry. Question about how quasicrystals grow still remains open. In this talk I will present the algorithm for growing defect free Penrose tilings using local rules based on paper "Growth rules for quasicrystals" by Joshua Socolar. And state necessary conditions for deterministic growth of cut-and-project aperiodic tilings using "defective seeds".

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Silvère Gangloff (Institut de Mathématiques de Toulouse):

## Information processing into hierarchical structures under dynamical constraints.

**Abstract:** Hochman and Meyerovitch (2010) and Meyerovitch (2011) gave characterizations of topological invariants of multidimensional subshifts using computability conditions. They used subshifts in which hierarchical structures appear into which are implemented some Turing machines used to verify frequency conditions on symbols of the subshift's alphabet. We propose adaptations to these results for subshifts under dynamical constraints : transitivity for the entropy characterization, and minimality for the entropy dimension.

## 

Edin Liđan (University of Bihać):

## Homology groups of generalized polyomino type tilings.

**Abstract:** A polyomino is a plane geometric figure formed by joining one or more equal squares edge to edge and it may be regarded as a finite subset of the regular square tiling with a connected interior. Polyomino tilling problem asks is it possible to properly cover a finite region M consisting of cells with polyomino shapes from a given set  $\mathcal{T}$ . There are a numerous generalizations of this questions towards symmetrical and asymmetrical tillings, higher dimension analogs, polyomino types in other regular lattice grids (triangular, hexagonal), etc. However, the problem in all cases in general is NP-hard and we can give definite answer only in limited number of cases.

This enthralling problem from recreational mathematics attracts attention of both mathematicians and non-experts. Conway and Lagarias devolped in [1] assigned to each set of tiles  $\mathcal{T}$  the homology and the homotopy group of tillings and formulated a necessary condition for existence of a proper tillings of a finite region M, and their ideas are further developed by Reid in [2]. This powerful idea allows natural generalization to a much wider class of combinatorial tillings. In the talk we study problem of tilling a surface S subdivided in finite 'combinatorial' grid which mail fail to be regular with finite set of polyomino like shapes  $\mathcal{T}$  and define the homology group  $H_S(\mathcal{T})$ . We present some new results together with illustrating examples explaining the application of the homology group of generalized polyomino type tillings in combinatorial and topological context. This is joint work with Dorde Baralić.

## References

- [1] J. H. Conway, J. C. Lagarias: *Tilings with polyominoes and combinatorial group theory*, Journal of Combinatorial Theory, Series A 53, (1990), 183 208.
- [2] M. Reid: *Tile homotopy groups*, L'Enseignement Mathématique 49 (2003), no. 1–2, pp. 123 155.

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Victor Lutfalla (University of Turku):

## Substitution cut-and-project tilings.

**Abstract:** In this talk I will present our work on substitution cut-and-project tilings with n-fold rotational symmetry. I will detail the case of 7-fold rotational symmetry and present our methodology for the study of 2k + 1-fold symmetry.

## **PARTICIPANTS**

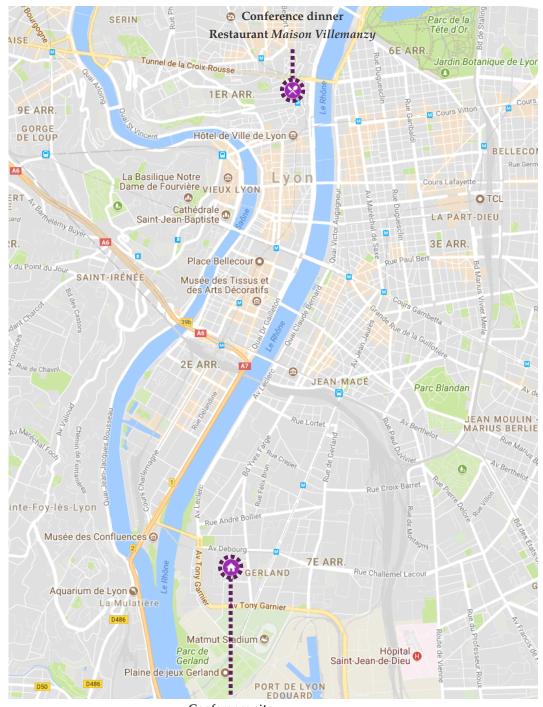
Mahmud AKELBEK, Weber State University Mauricio Alfredo Allendes Cerda, University of Chile Nathalie AUBRUN, Florent BECKER, Université d'Orléans Jérémie Bouttier, ENS de Lyon Mikael de la salle, CNRS, ENS de Lyon Julien Esnay, Thomas Fernique, CNRS & Paris 13 Dominik FRANCOEUR, University of Geneva Ilya Galanov, Paris 13 Guilhem GAMARD, Higher School of Economics, Moscow Silvère GANGLOFF, Université Paul Sabatier Thibault GODIN, Paris 7 Diderot Anael GRANDJEAN, Université de Montpellier Pierre Guillon, CNRS Bin HAN, Institut Camille Jordan Joonatan JALONEN, University of Turku Jarkko KARI, University of Turku Johannes Kellendonk, Paul-Henry LEEMANN, ENS Lyon Edin LIĐAN, University of Bihać Victor LUTFALLA, Alba Marina Málaga Sabogal, INRIA de Paris Diana MANDAR, Etienne Moutot, ENS Lyon Samuel NARTEY, Moscow Institute of Physics and Technology MUSTAFA OZKARACA, Daria PCHELINA, Aitor Perez, Matthieu PICANTIN, Université Paris Diderot Michaël RAO, Eric REMILA, Andrew Ryzhikov, Université Grenoble Alpes Lorenzo SADUN, Ilkka Törmä, University of Turku Siming Tu, University of Chile

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## Social event

## Conference dinner at Maison Villemanzy

The conference dinner takes place on Wednesday, September 20th at 19:30, at the restaurant **Maison Villemanzy**,

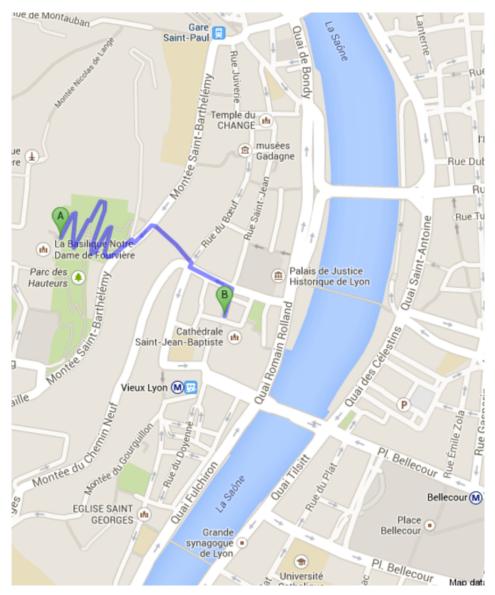


Conference site 1 place de l'école, ÉNS de Lyon (site Monod)

#### Useful information and tourism

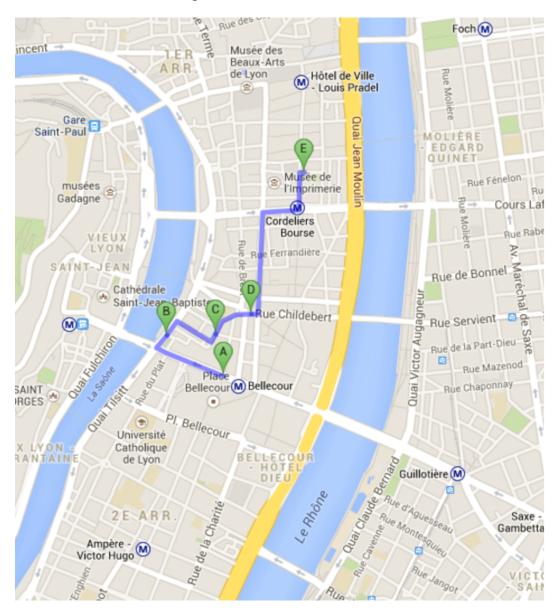
## Climbing to the Fourvière basilica

You can discover the most famous church of Lyon, the basilica *Notre Dame de Fourvière*, by a small itinerary given below. From the subway station *Vieux Lyon*, take the funicular railway to the *Fourvière* hill. Once arrived at the top of the hill, you can visit the *Fourvière basilica* (point A on the map) and enjoy a nice view of the whole city. Then you can go back to *Vieux Lyon* and to the *Saint Jean cathedral* (point B on the map) by going down through the garden (you can also take the garden to climb to the basilica, but the bottom entry of the garden is not easy to find, and obviously, it is always easier to go down than to climb up). The map given below can also be found at: http://tinyurl.com/paltmwy Duration: around 45 minutes starting from the subway station *Vieux Lyon*.



## WALKING IN THE PRESQU'ILE

The *Presqu'île*, the city center located between the two rivers the *Saône* and the *Rhône*, can be discovered by a small walk like the one given on the map below. By this itinerary, you will successively go through the *place Bellecour* (point A on the map), the docks of the *Saône* river (point B), the *théâtre des Célestins* (point C), the *place des Jacobins* (point D), the street *rue de la République* (with a lot of shops), and you will finally arrive at the *place de la Bourse* (point E). Once arrived, you can go to the street called *rue Mercière*, near the *place de la Bourse*, where you can find a lot of restaurants. The map given below can also be found at: http://tinyurl.com/npmaaf3



Duration: around 35 minutes starting from the *Place Bellecour*.

#### Some other typical streets and neighbourhoods of Lyon

- The neighbourhood of *Vieux Lyon* and the *quartier Saint Jean*
- The street *rue des Marronniers* (a walking street with restaurants)
- The street *rue Mercière* (a street with a lot of restaurants)
- The docks of the *Rhône* river (by walk or by bike)
- Streets around *place des Terreaux*, like *rue Sainte Catherine* or *rue Pizay* (small streets with pubs)
- The *Parc de la Tête d'Or* (a free parc with a zoo)

### **Restaurants in Lyon**

Most of the restaurants and bars in LYON are located in the old city center and in the *Presqu'île* (1st and 2nd arrondissements). We propose below few restaurants near the conference site. For more suggestions or restaurants in the old city center, please check the online list from the tourist office which can be found at: www.lyon-france.com/Restaurants

RESTAURANTS NEAR THE CONFERENCE SITE (see map p. 12):

- Ninkasi-Le Silo, 267 Rue Marcel Mérieux, 69007 Lyon (around 20€)
- René Gamboni, ,241 Rue Marcel Mérieux, 69007 Lyon (around 30-35€)
- Carnegie Hall, 253 rue Marcel Mérieux, 69007 Lyon (mainly meat, around 30-35€)
- Jols, 283 Avenue Jean Jaurès, 69007 Lyon (mainly fish and seafood, around 30-35€)

### Coming to the conference site

The closest airport is *Lyon Saint-Exupéry* (remember The Little Prince by Antoine de Saint-Exupéry? Saint-Exupéry was from Lyon!). There is also a TGV train station near the airport. The tramway takes you directly to one of Lyon downtown train stations, *Lyon Part-Dieu*. From there you can take the subway B for a short ride to the conference site.

**Be careful:** there are **two different sites** for the ÉNS Lyon, **site Monod** for sciences and site Descartes for litterature. The *Tilings in Lyon events* take place at the **site Monod**.

## How to get to the ÉNS Lyon, site Monod, 1 place de l'école

- From the Lyon-Saint Exupéry Airport: take the Rhône Express from the airport to the Part-Dieu train station.
- From the Lyon Part-Dieu train station: take the Vivier Merle exit of the train station, then take the subway, line B in the direction of *Gare d'Oullins*. Get down at the *Debourg* stop (see below in green).
- From the Lyon Perrache train station, by bus: go to the bus station and take the bus line C22. Get down at the *ÉNS Lyon* stop (see below in red).
- From the Lyon Perrache train station, by tramway: take the tramway line T1 at the station *Perrache* or at the outdoor tramway station *Suchet*. Get down at the *ÉNS Lyon* stop (see below in red).
- **From anywhere by** *Vélov'* **bike**: you can use the city bikes system, called *Vélov'*. See the website of Vélov' for details (how to rent a bike, prices and delay, stations with available bikes...).

