

Homology groups of generalized polyomino type tilings

Edin Liđan

University of Bihać
Bihać, Bosnia and Herzegovina

Abstract

A polyomino is a plane geometric figure formed by joining one or more equal squares edge to edge and it may be regarded as a finite subset of the regular square tiling with a connected interior. Polyomino tiling problem asks is it possible to properly cover a finite region M consisting of cells with polyomino shapes from a given set \mathcal{T} . There are a numerous generalizations of this questions towards symmetrical and asymmetrical tilings, higher dimension analogs, polyomino types in other regular lattice grids (triangular, hexagonal), etc. However, the problem in all cases in general is NP-hard and we can give definite answer only in limited number of cases.

This enthralling problem from recreational mathematics attracts attention of both mathematicians and non-experts. Conway and Lagarias devolved in [1] assigned to each set of tiles \mathcal{T} the homology and the homotopy group of tilings and formulated a necessary condition for existence of a proper tilings of a finite region M , and their ideas are further developed by Reid in [2]. This powerful idea allows natural generalization to a much wider class of combinatorial tilings. In the talk we study problem of tiling a surface S subdivided in finite ‘combinatorial’ grid which mail fail to be regular with finite set of polyomino like shapes \mathcal{T} and define the homology group $H_S(\mathcal{T})$. We present some new results together with illustrating examples explaining the application of the homology group of generalized polyomino type tilings in combinatorial and topological context. This is joint work with Đorđe Baralić.

References

- [1] J. H. Conway, J. C. Lacarias: *Tilings with polyominoes and combinatorial group theory*, Journal of Combinatorial Theory, Series A 53, (1990), 183 – 208.
- [2] M. Reid: *Tile homotopy groups*, L’Enseignement Math’ematique 49 (2003), no. 1–2, pp. 123 – 155.