Continuous eigenvalues for Meyer sets.

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Abstract

Let $D \subset \mathbb{R}^d$ a Delone set and $(X_D, \mathbb{R}^d)$ its dynamic hull system. Following [1] when the abelian group $[D - D]$ is a finitely generated with r generators, we say that the rank of D is r and we write $rank(D) = r$. In this talk we will give a dynamical proof of the following result that appears in [3].

Theorem 1. *Let $D \subset \mathbb{R}^d$ be a repetitive Meyer set with $rank(D) = r$. The system $(X_D, \mathbb{R}^d)$ has $r \geq d$ continuous eigenvalues.*

In dimension $d = 1$, following some ideas in [2], we will give conditions that ensure that every eigenvalue is continuous.

Theorem 2. *Let $D \subset \mathbb{R}$ be a repetitive Meyer set such that $(X_D, \mathbb{R})$ is linearly recurrent. Suppose that exists a sequence of Kakutani-Rokhlin partitions such that for any integer $m \geq 1$ the heights $h_1(m), \ldots, h_{c(m)}(m)$ are rationally independent. Then $(X_D, \mathbb{R})$ has only continuous eigenvalues.*

We will exhibit some examples of dynamical system where Theorem 2 applies. Also we work two examples to observe that the hypotheses of being Meyer and having rationally independent heights are necessary in Theorem 2.

This is a joint work with Daniel Coronel.

References:

[1] J. C. Lagarias. *Geometric models for quasicrystals I. Delone sets of finite type.* Discrete Comput. Geom. 21 (1999), no. 2, 161-191. MR 1668082.

[2] Cortez, Maria Isabel; Durand, Fabien; Host, Bernard; Maass, Alejandro. *Continuous and measurable eigenfunctions of linearly recurrent dynamical Cantor systems.* J. London Math. Soc. (2) 67 (2003), no. 3, 790-804.

[3] Johannes Kellendonk; Lorenzo Sadun. *Meyer sets, topological eigenvalues, and Cantor fiber bundles.* J. Lond. Math. Soc. (2) 89 (2014), no. 1, 114-130.