Edmund Harriss, University of Arkansas
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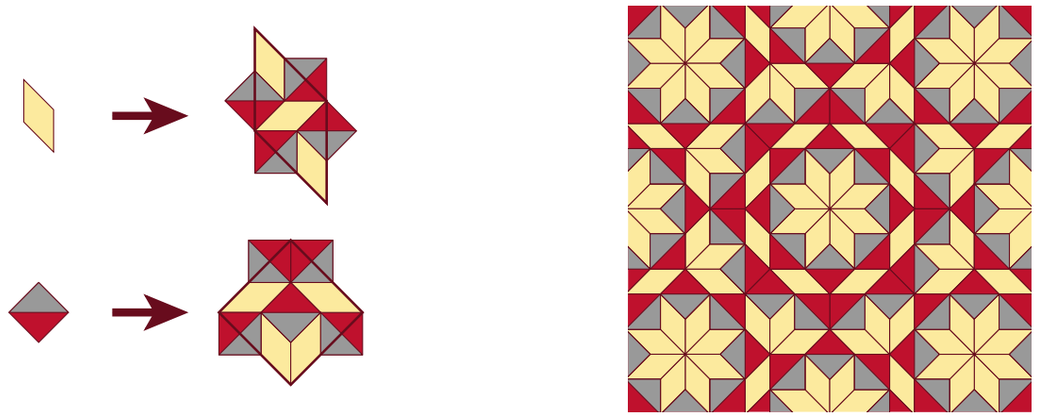
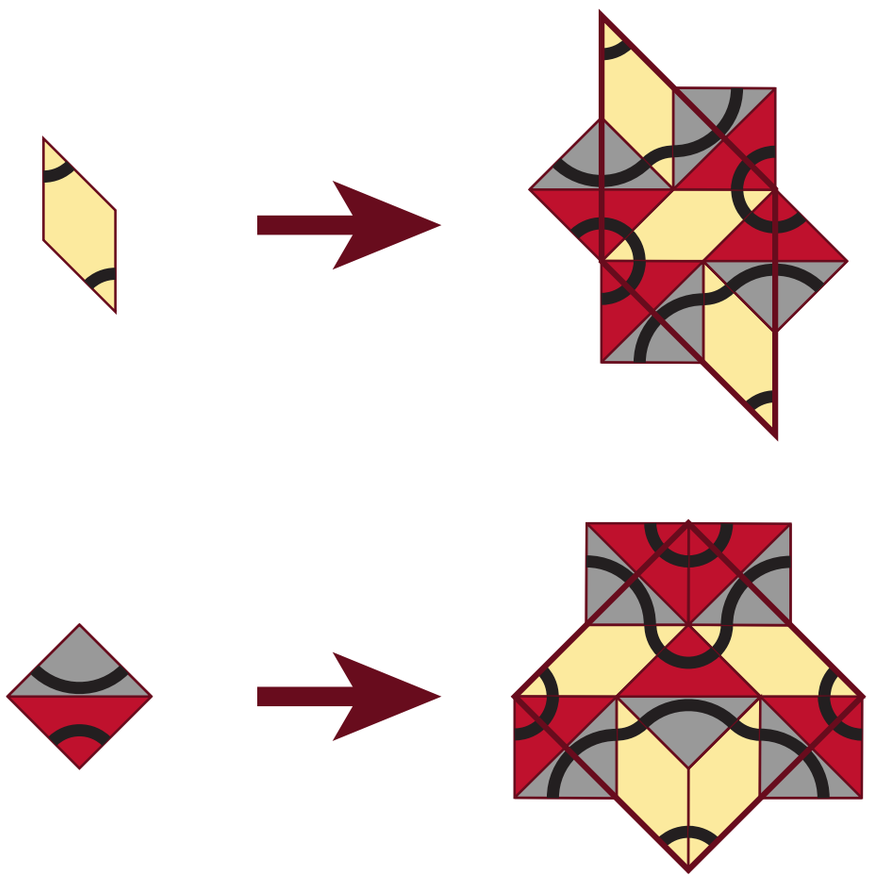
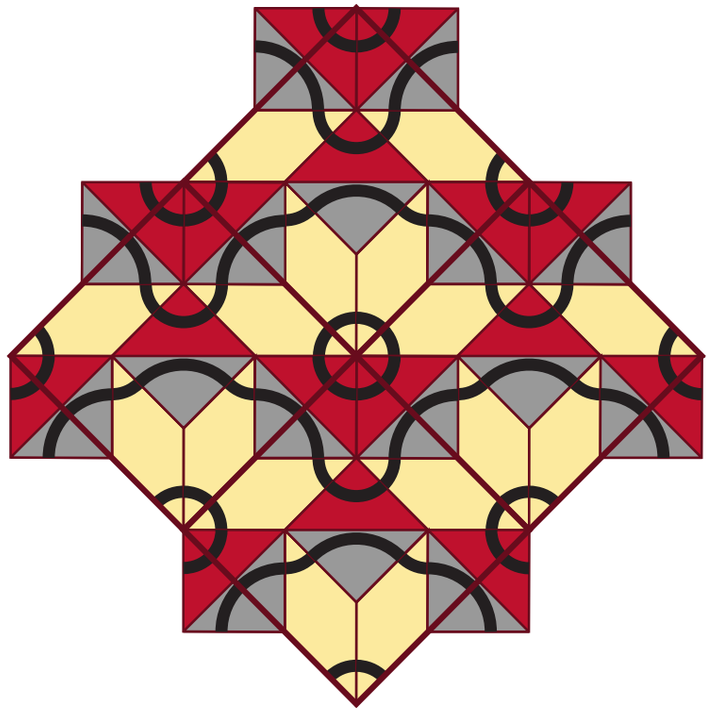
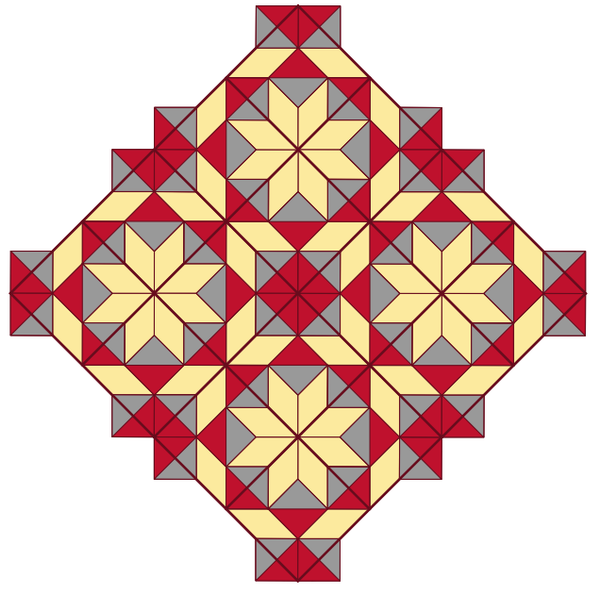
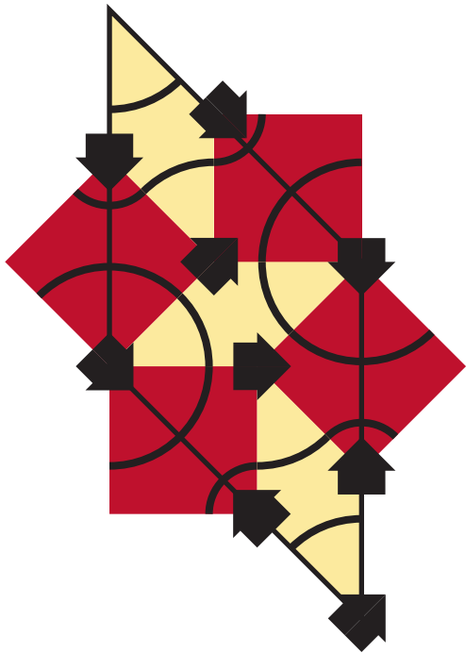
Matching rules for Hierarchy Lecture 2

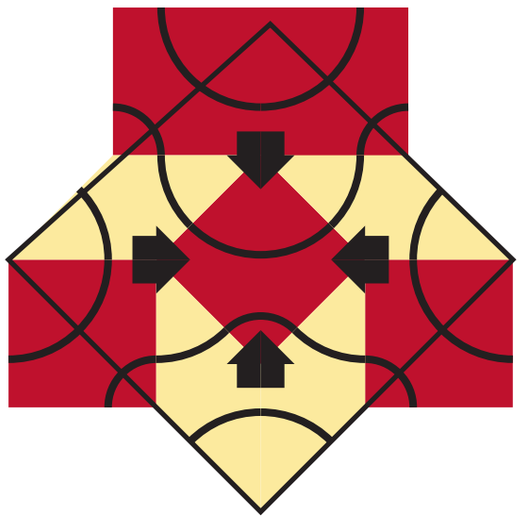


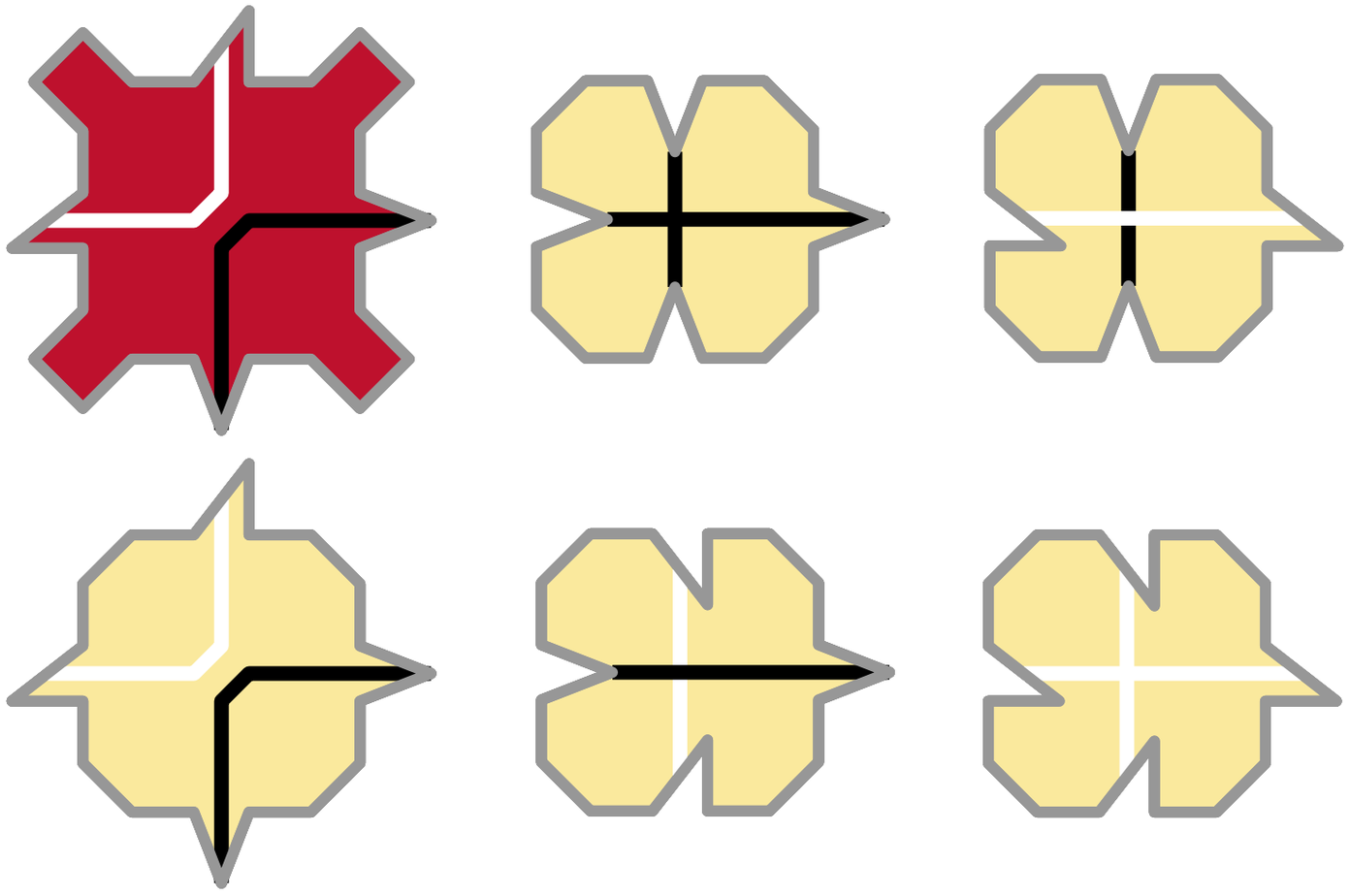


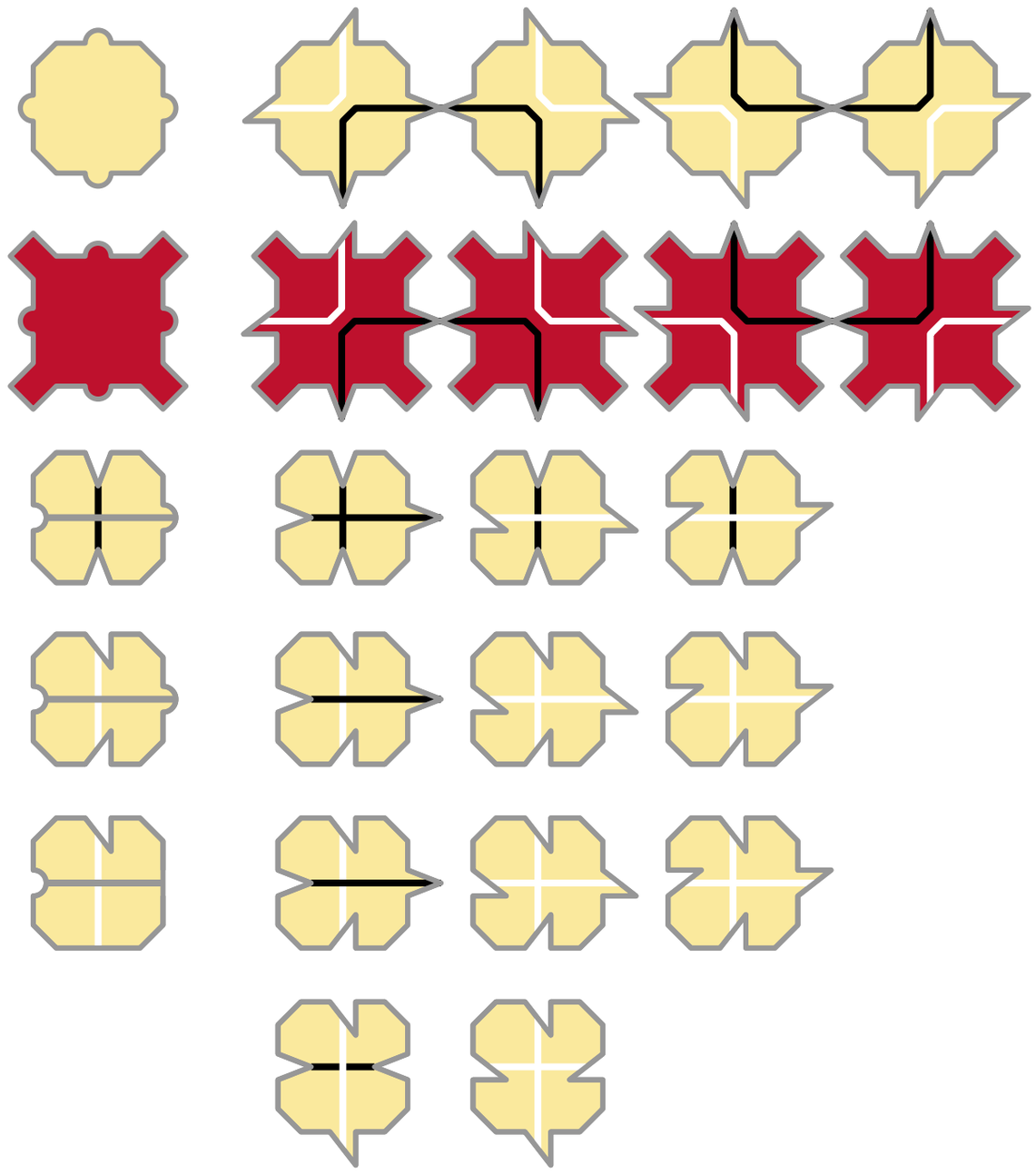


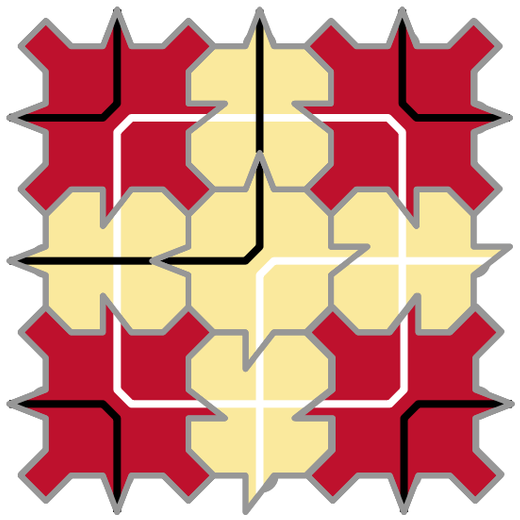


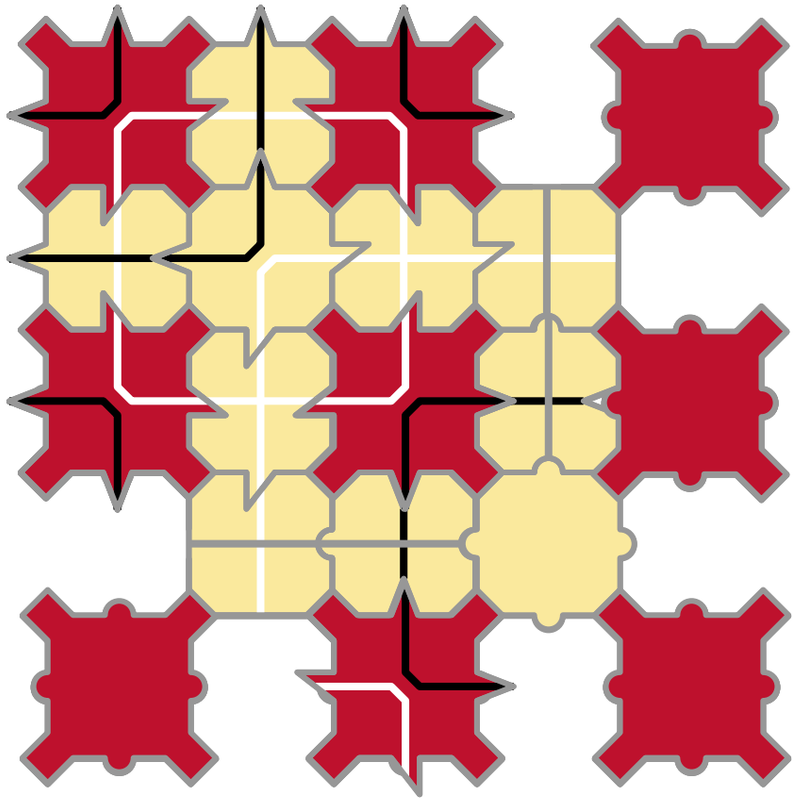


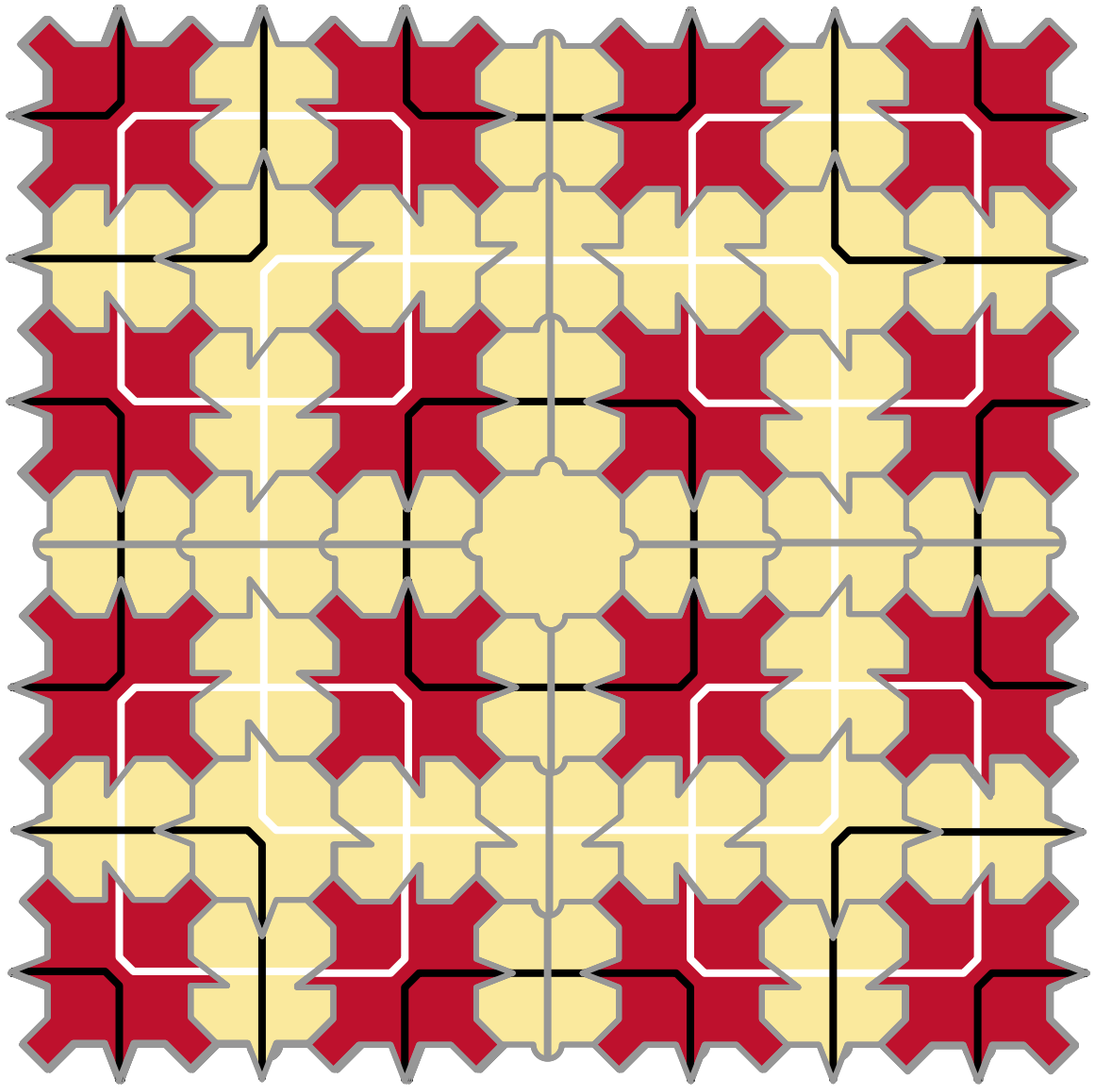


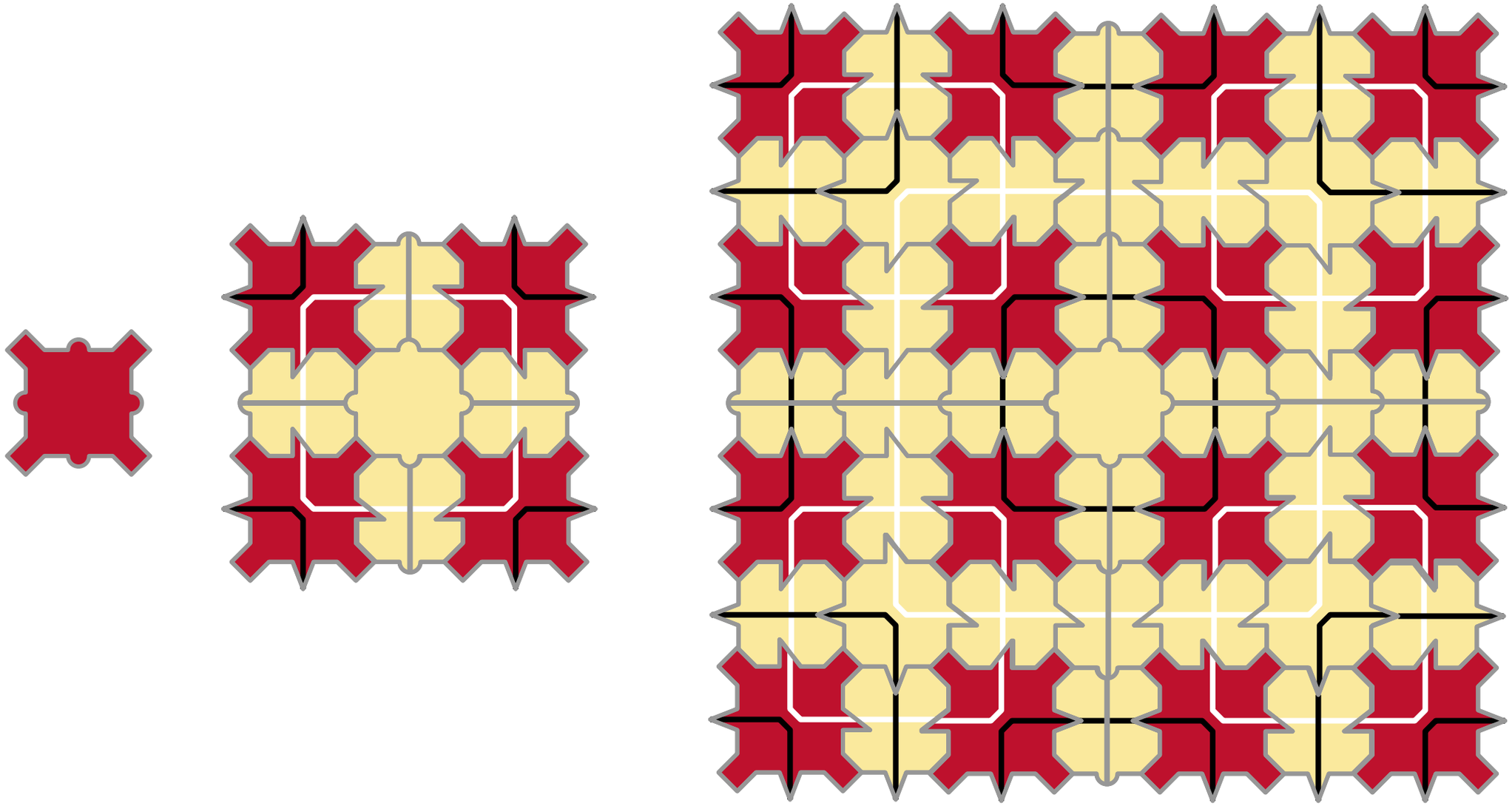


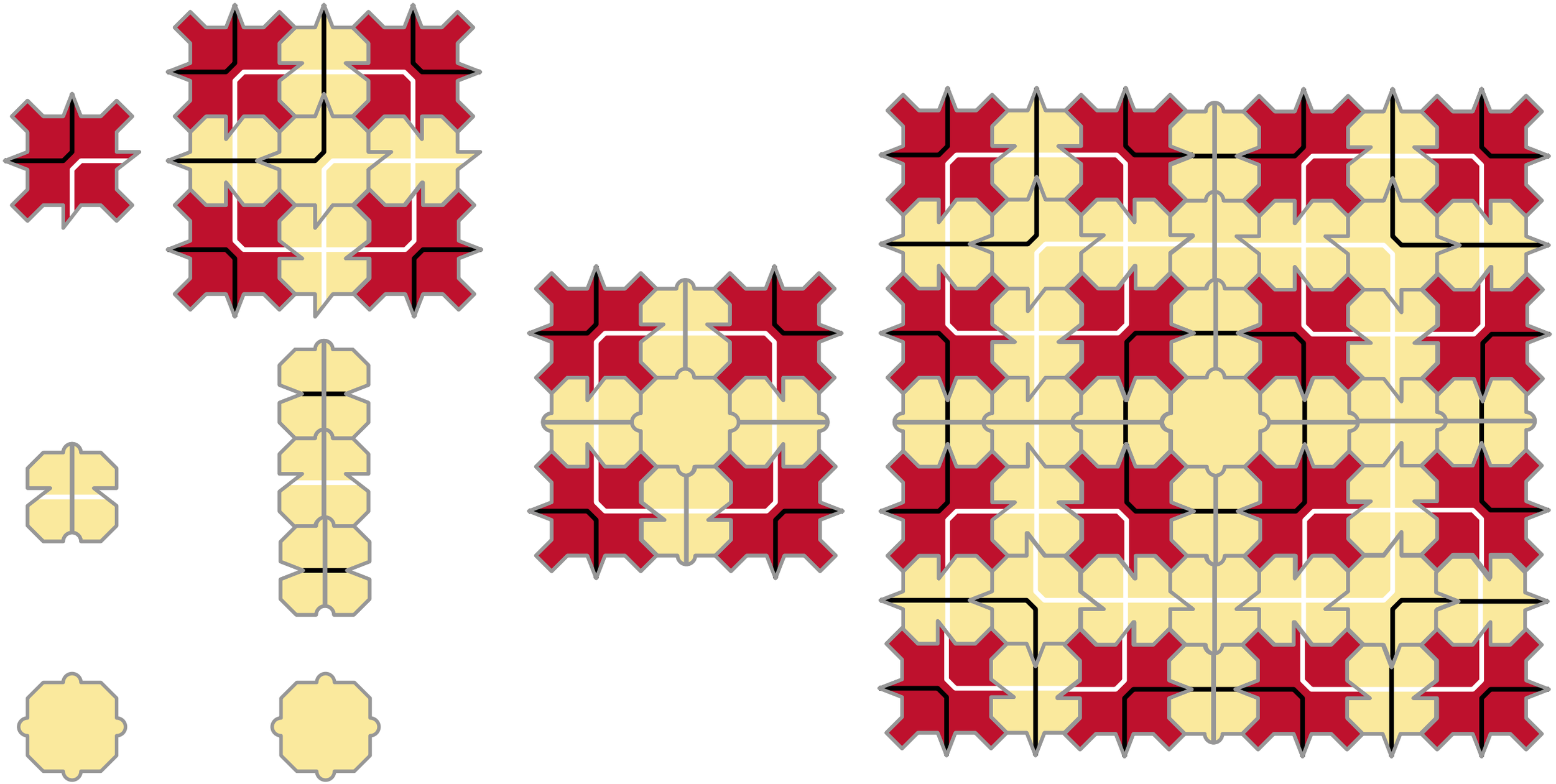












Key Ideas:

Tiles can pass information to coordinate structure.

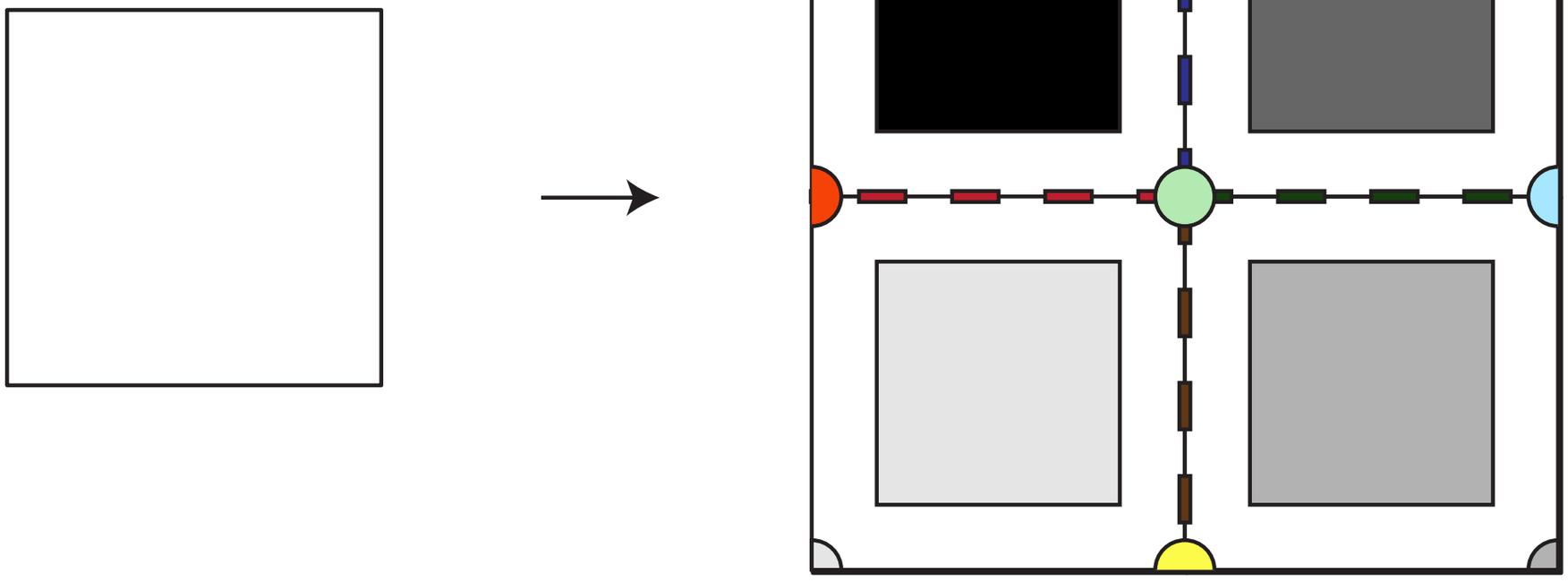
Edge structures can get longer, passing information without knowing when it will be used.

Some marking enforce coordination.

Start with the simplest possible substitution rule...

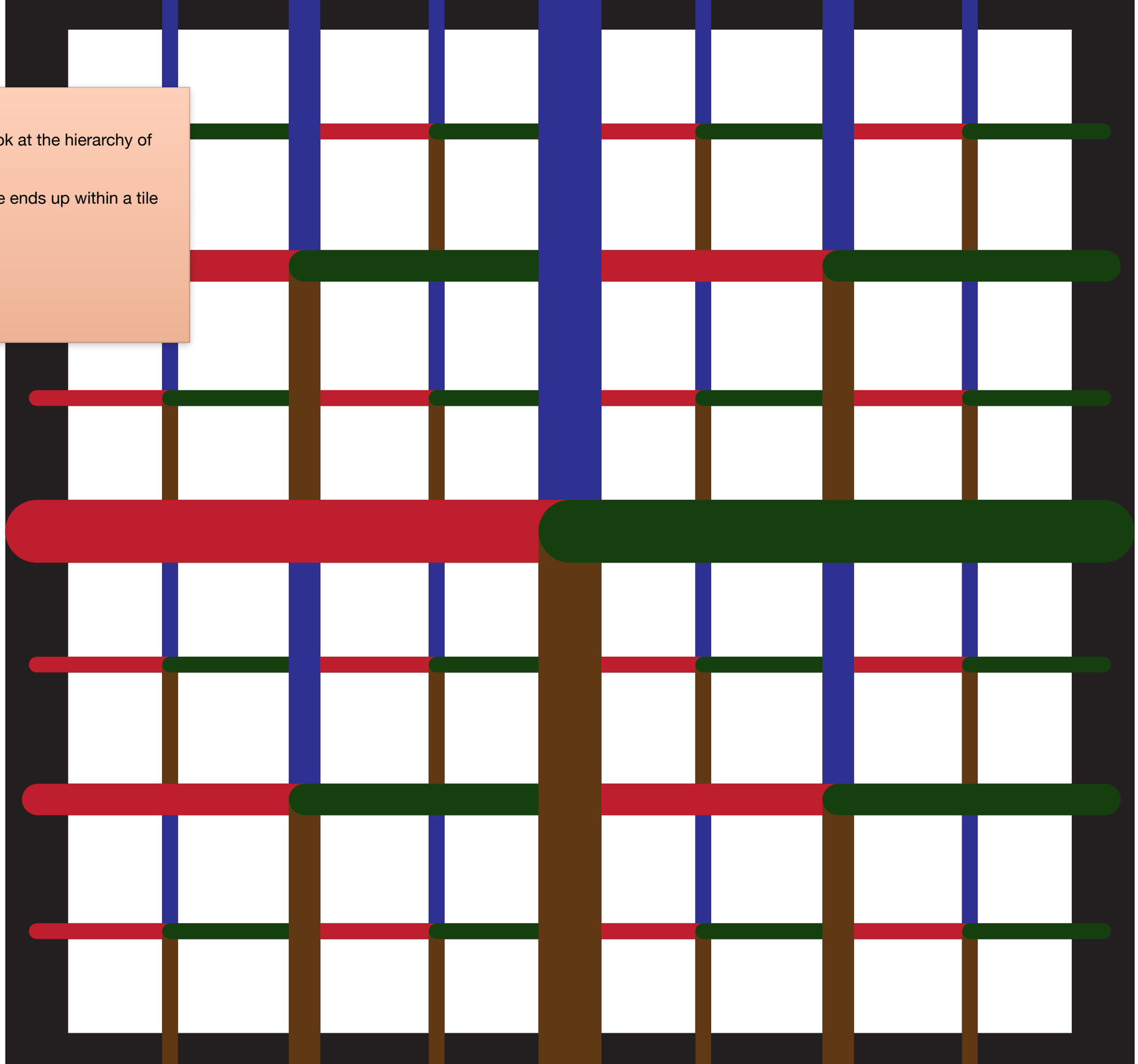
Label some features:

Edges, Vertices, Tiles



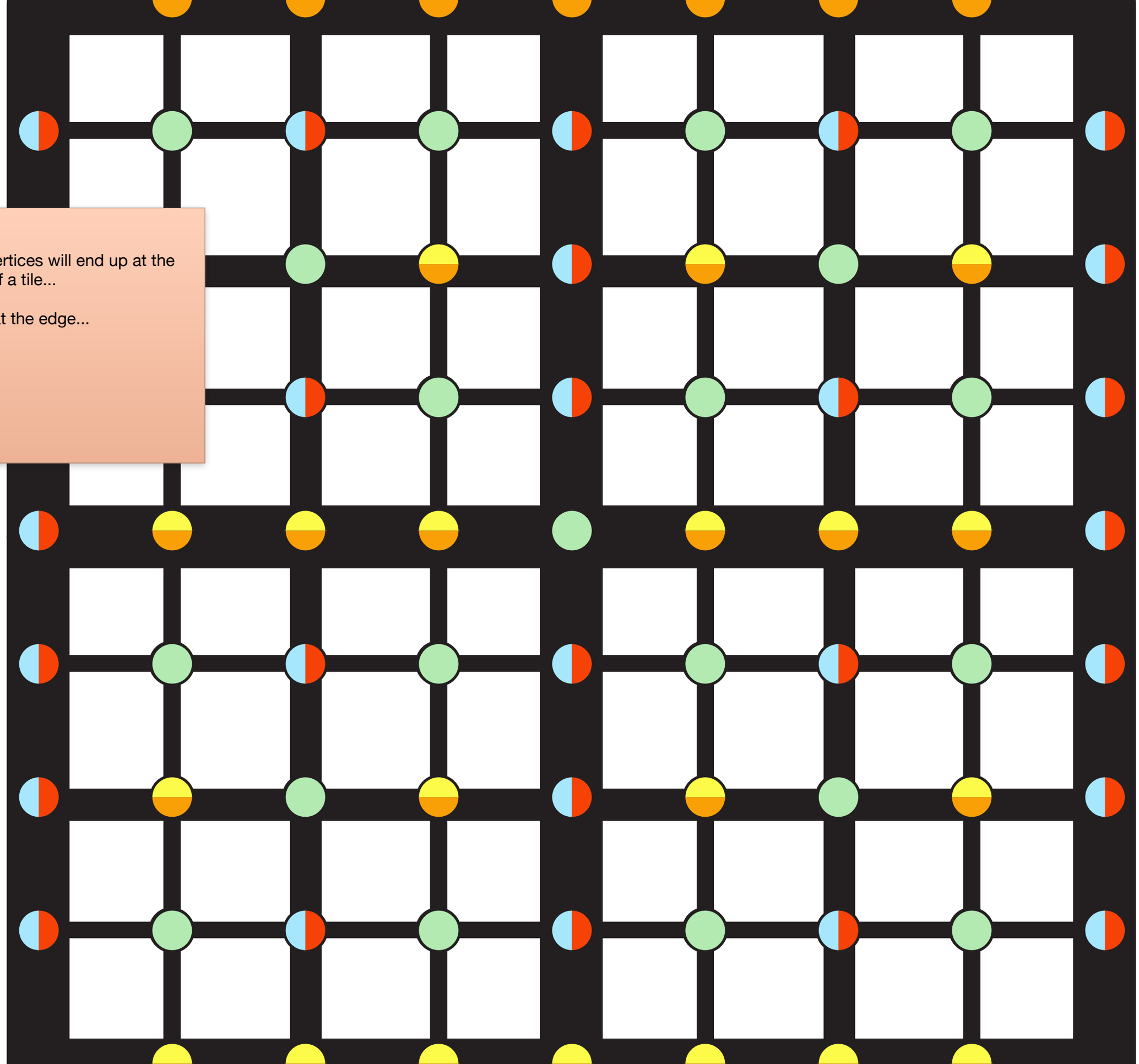
We can look at the hierarchy of the tiling.

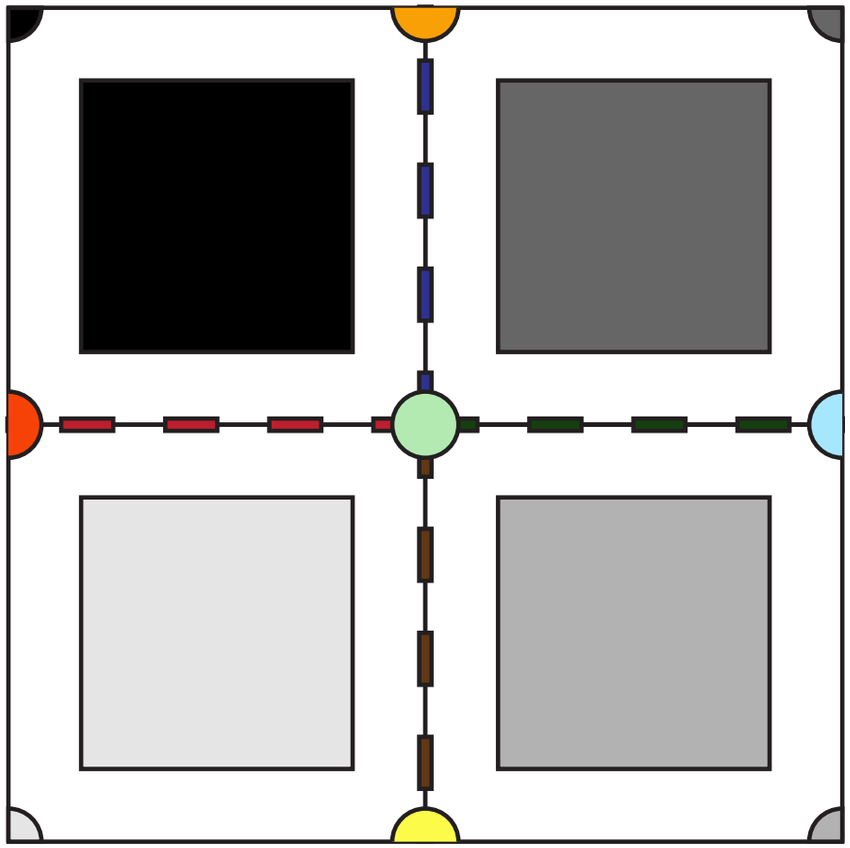
Every edge ends up within a tile



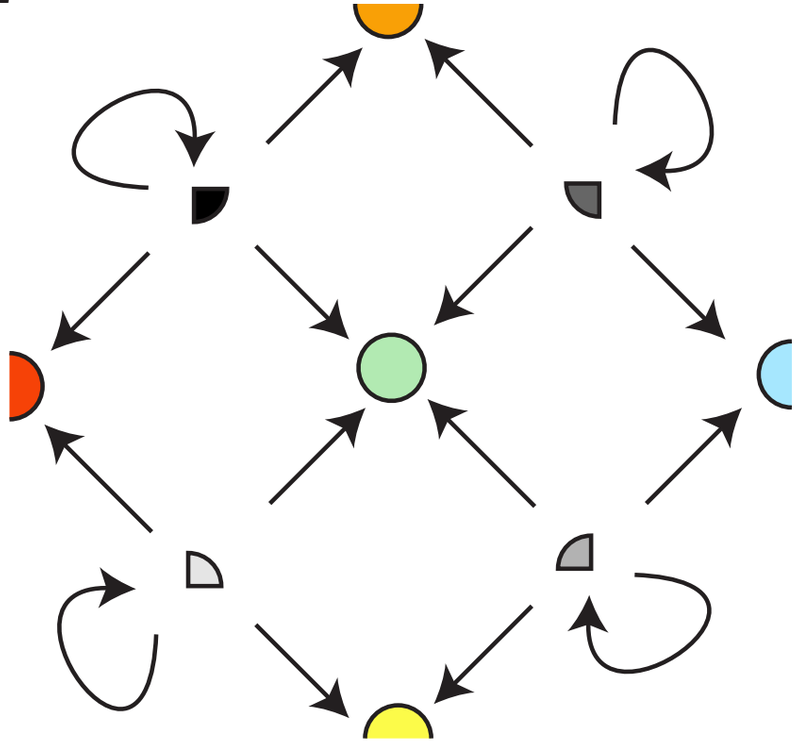
Some vertices will end up at the centre of a tile...

Others at the edge...





We can build a graph to show the possible roles that a vertex can take.



Every Tile knows:

Its tile type
The eventual type of its special vertex

Every Edge knows:

Its eventual type
What supertile it lies in:

The tile type
The eventual type of its special vertex

Every Vertex knows

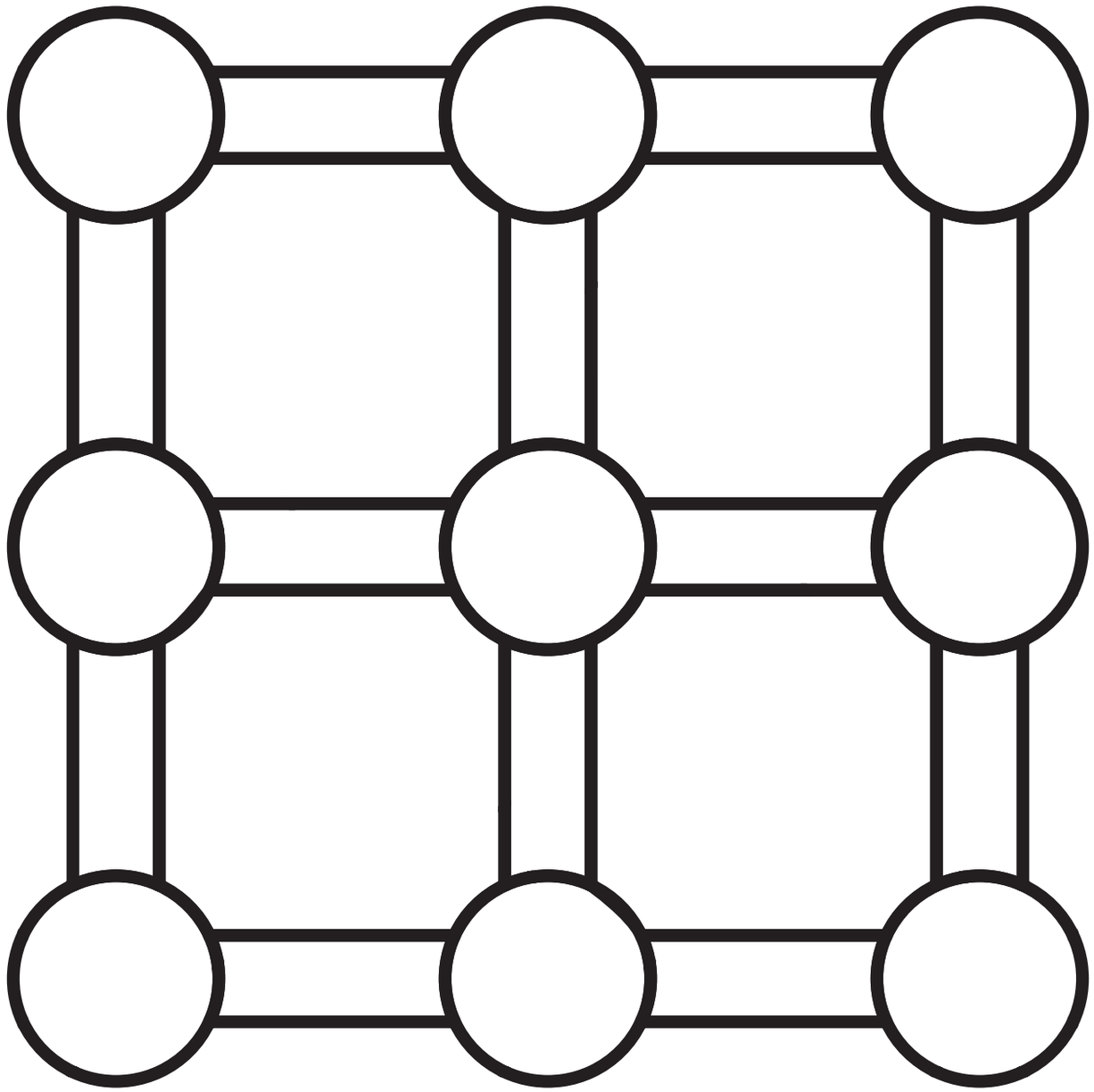
Its eventual type
What edges join it
What supertile it lies in:

The tile type
The eventual type of its special vertex

We want this information on the objects. The key is edges, they can grow transporting the information around the tiling.

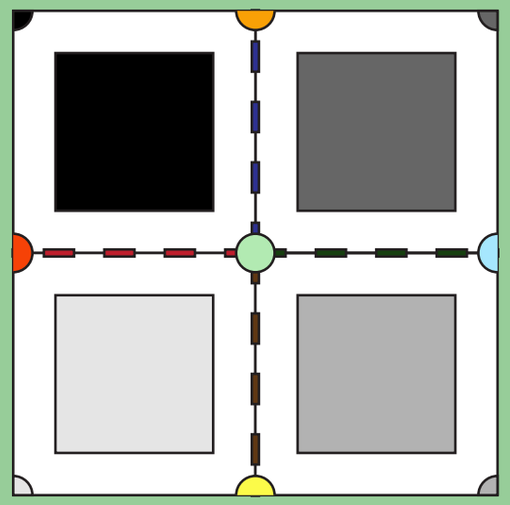
Now

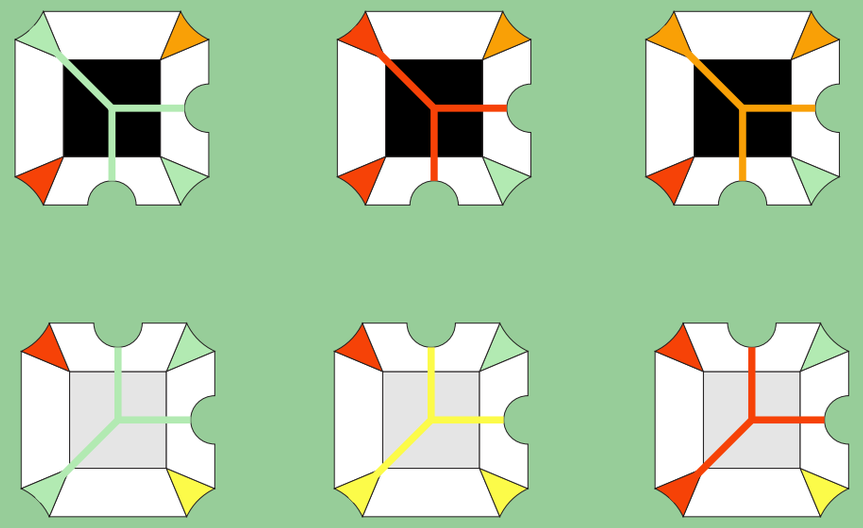
The tiles can be cut up to give the edges and vertices shape

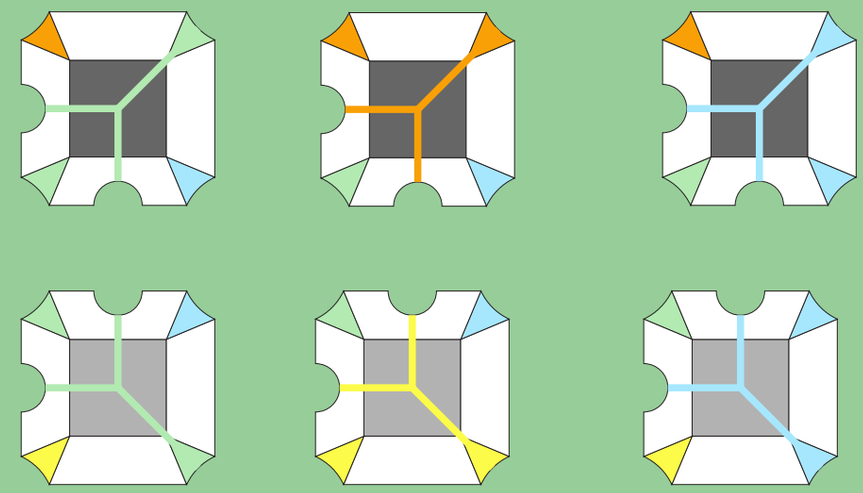


We can start with the tiles. Each knows its type and the type of its special vertex.

The edges of the supertile will also need to know the type of the special vertex, so the information is passed up to the internal edges.

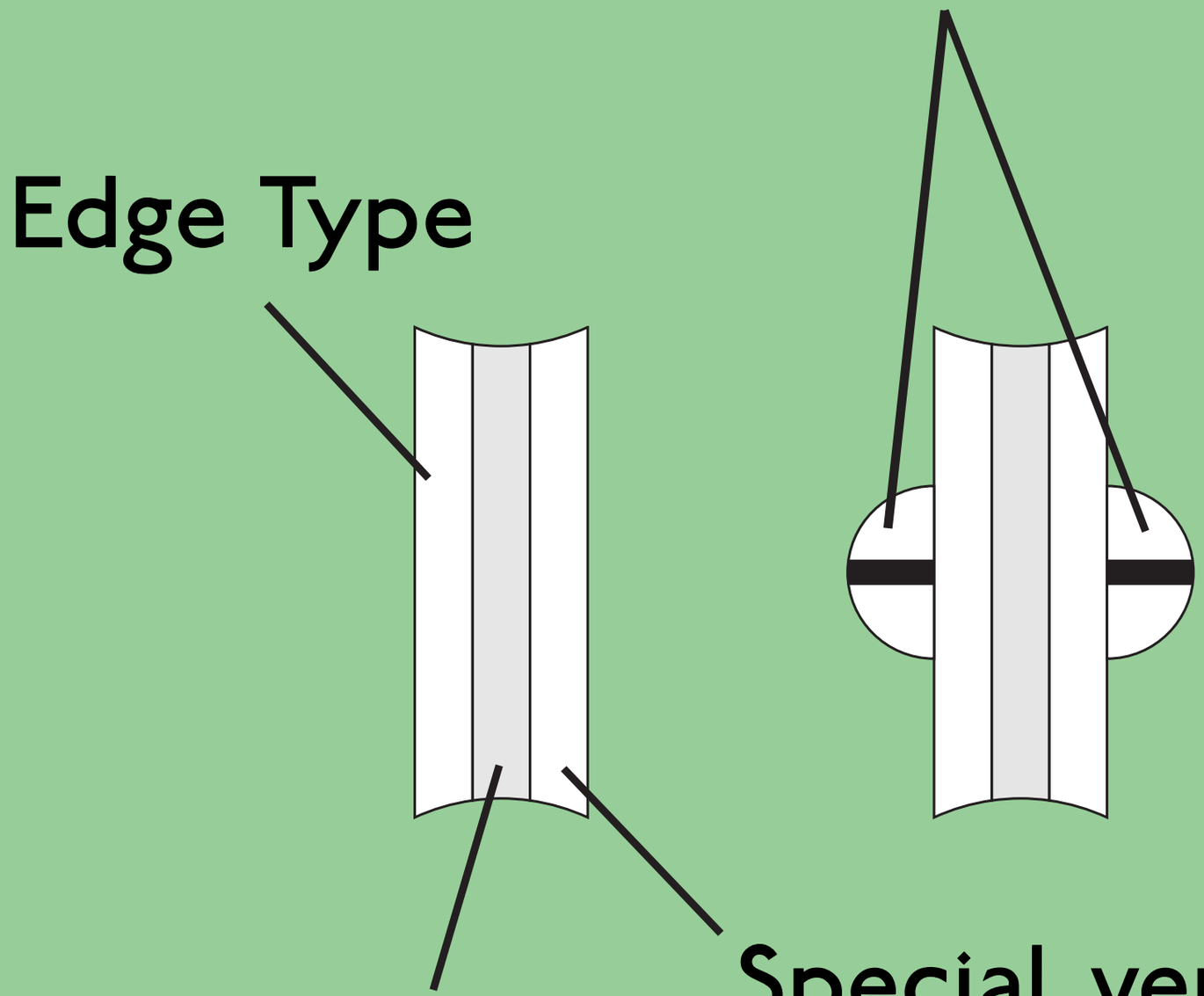






Tile special vertext type

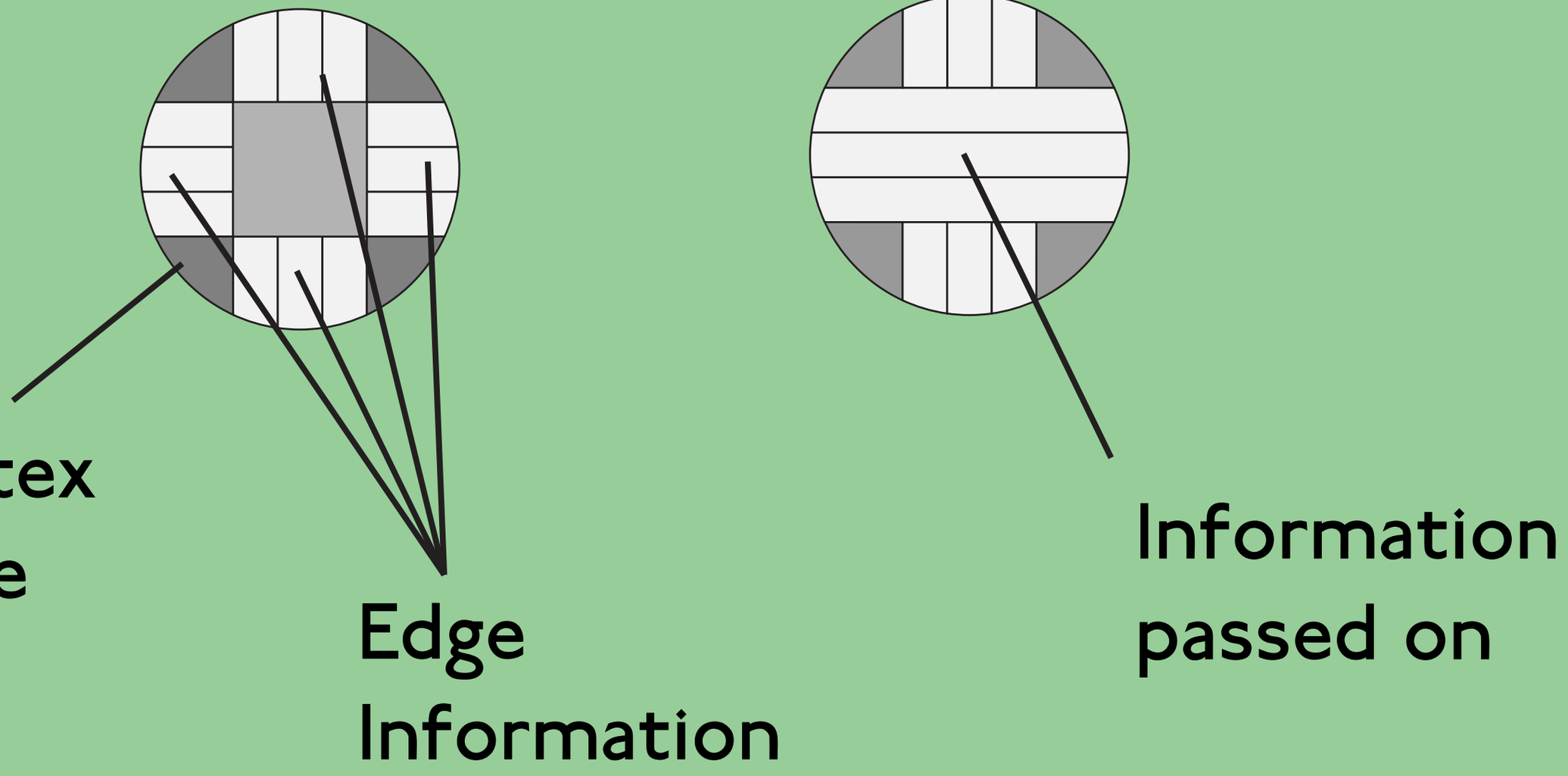
Edge Type



Supertile Type

Special vertex Type

Now look at edges. Each edge has three channels for the information it carries. There are also edges that plug into tiles.


Vertex
Type
Edge
Information
Information
passed on

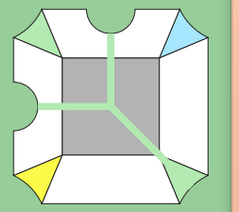
Lets build up a patch of tiling...

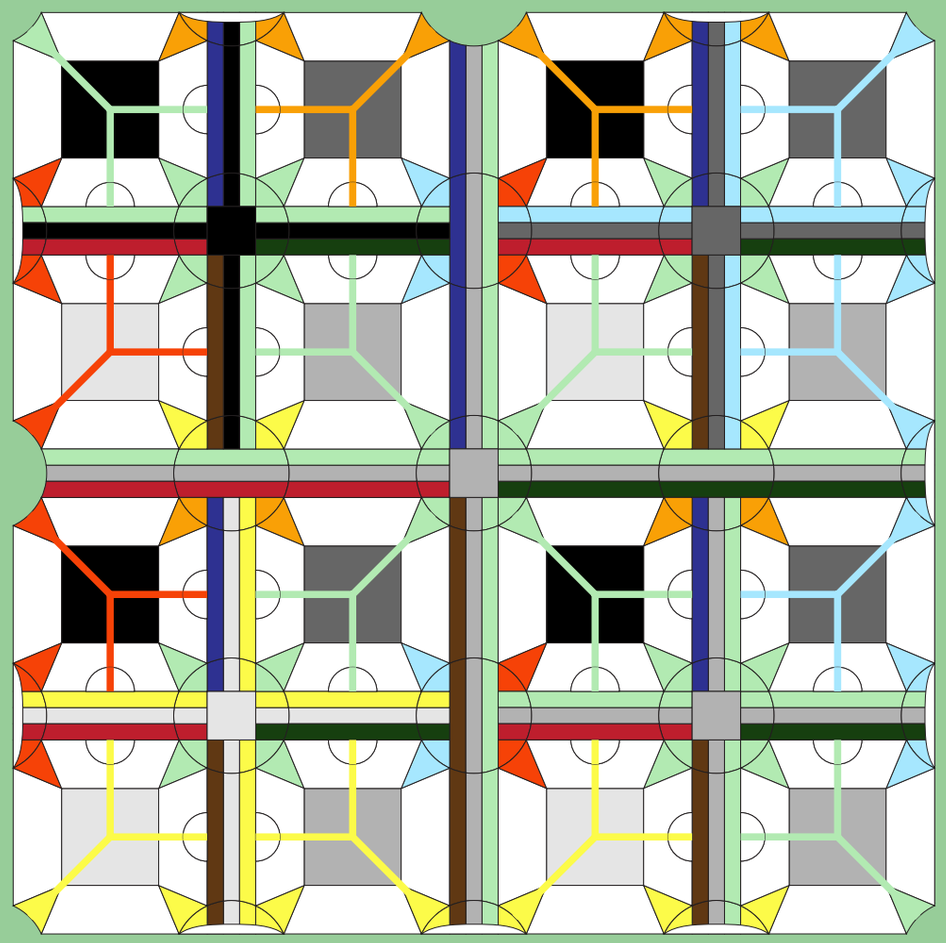
Note how the special vertex type is communicated up the hierarchy.

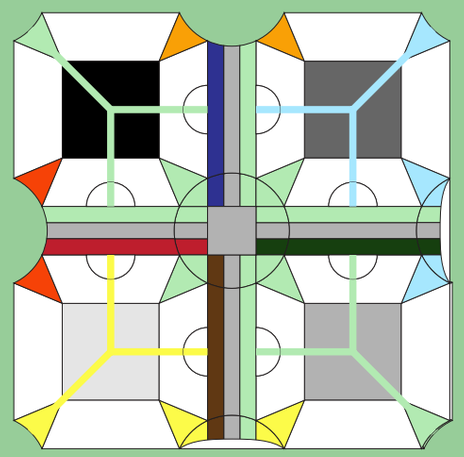
Thus each eleemnt can have finite information so there are a finite number of tiles...

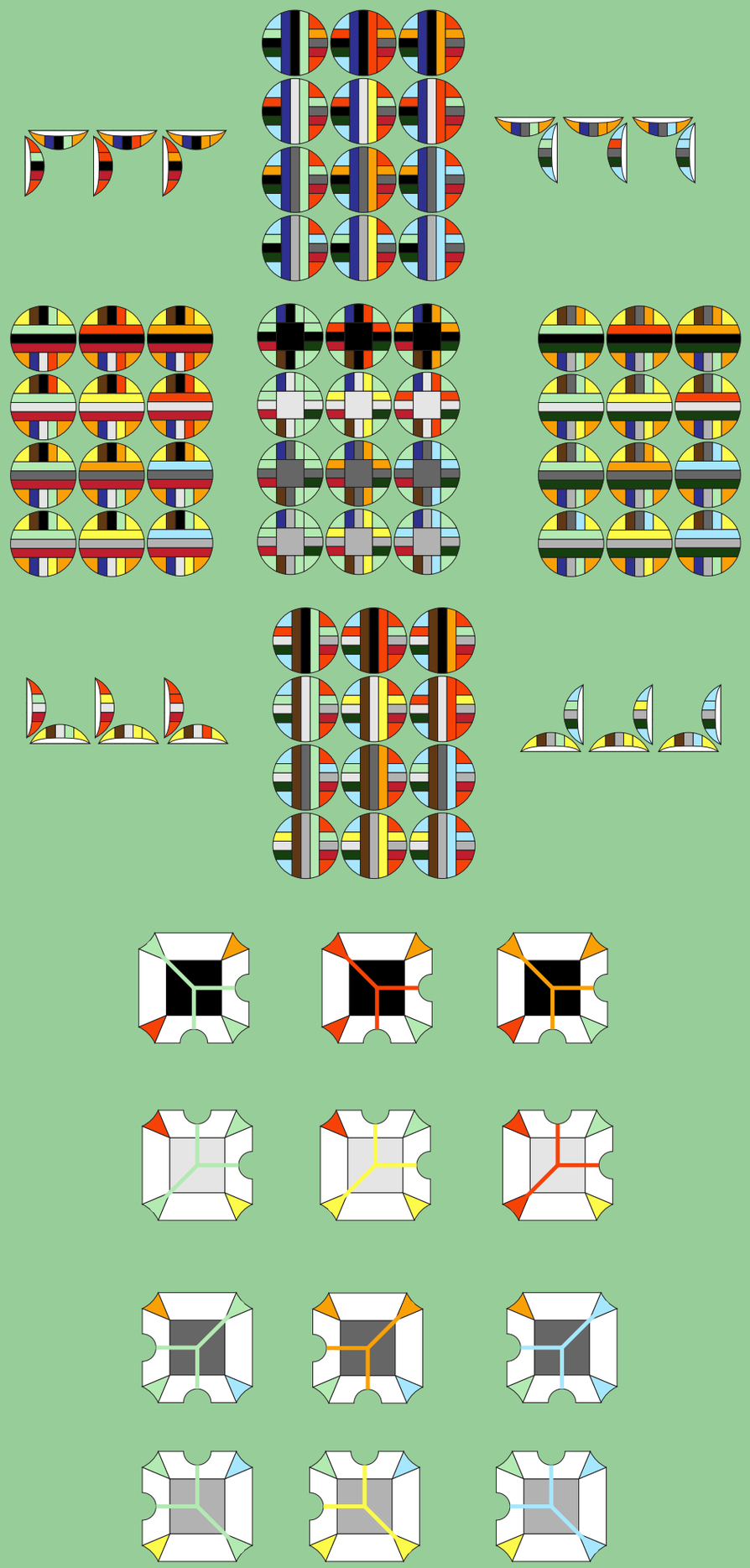
but...

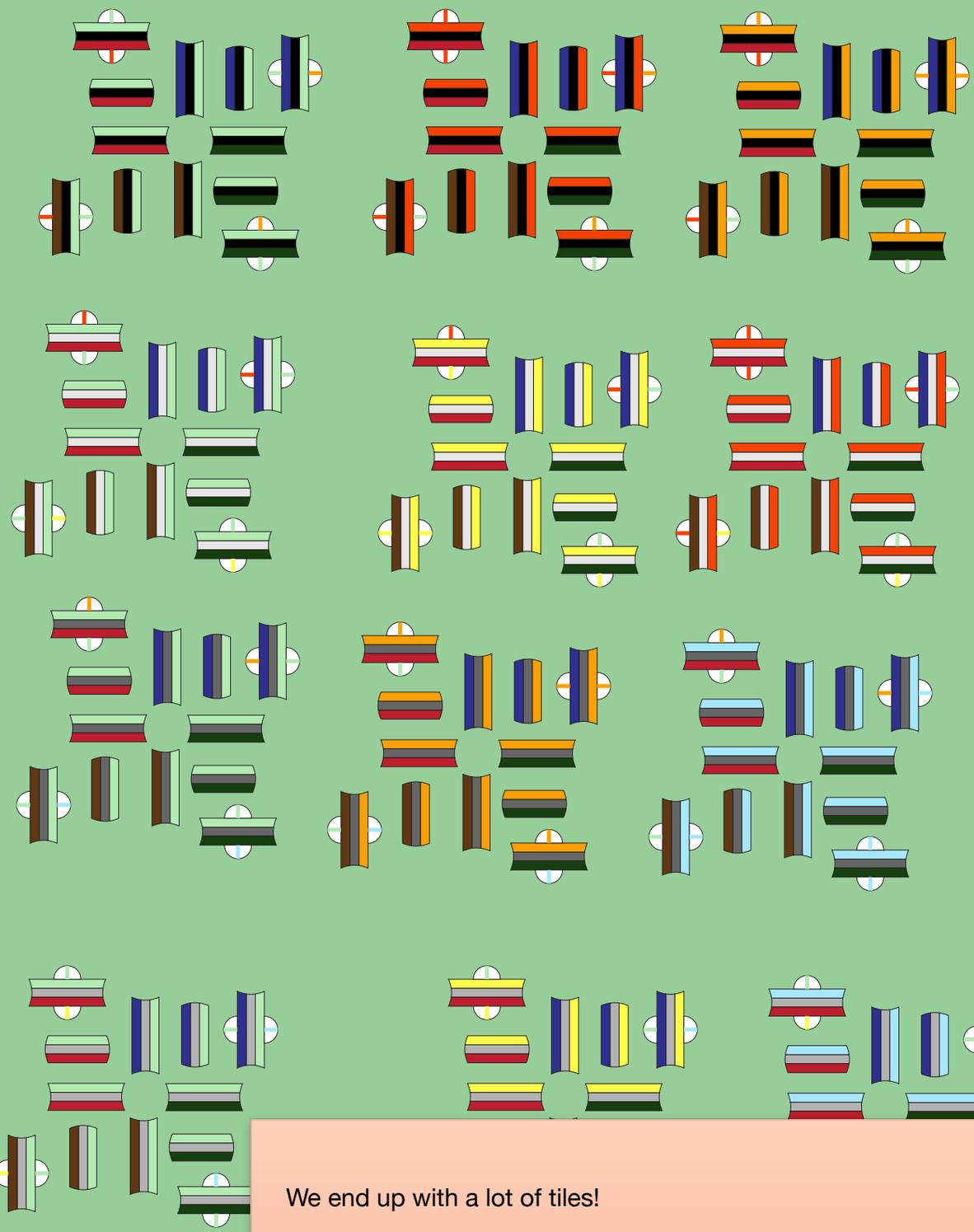
there are quite a few choices so...







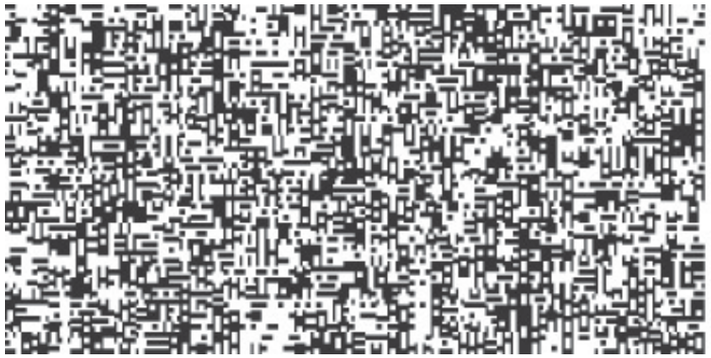


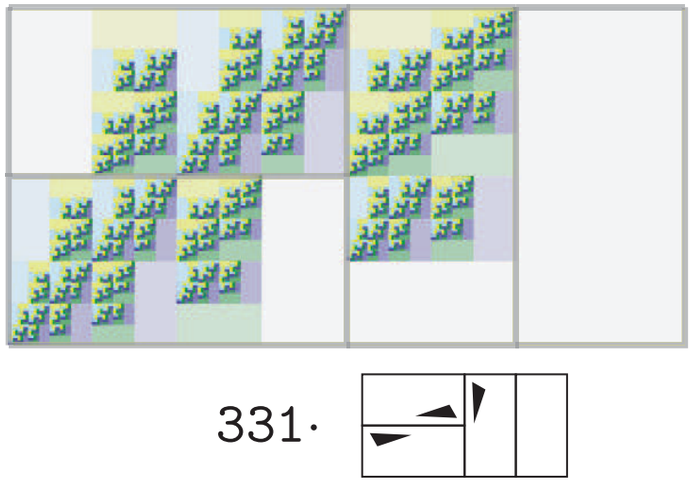


We end up with a lot of tiles!

The nice thing is that the information that travels round is explicit.

All the interactions are local, yet some information is forced to travel arbitarily far. Something I at least find amazing.





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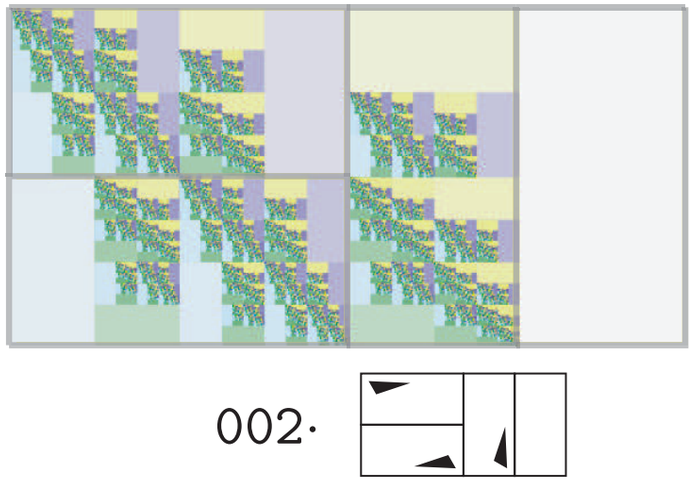

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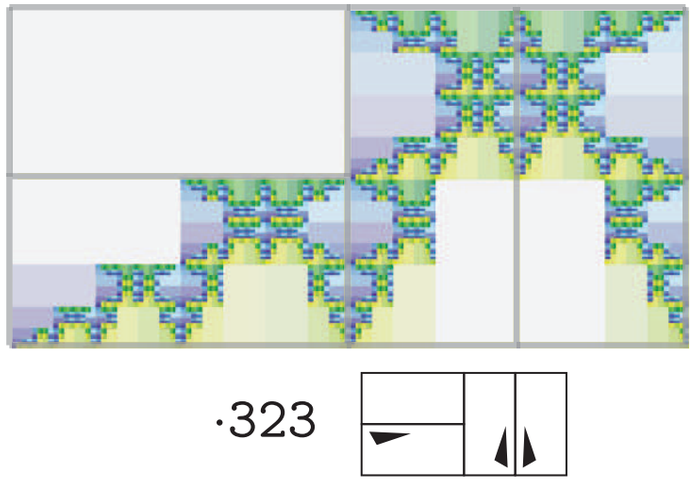

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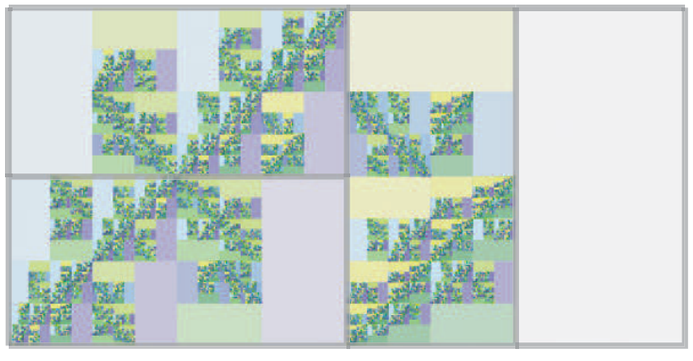

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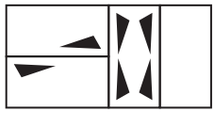

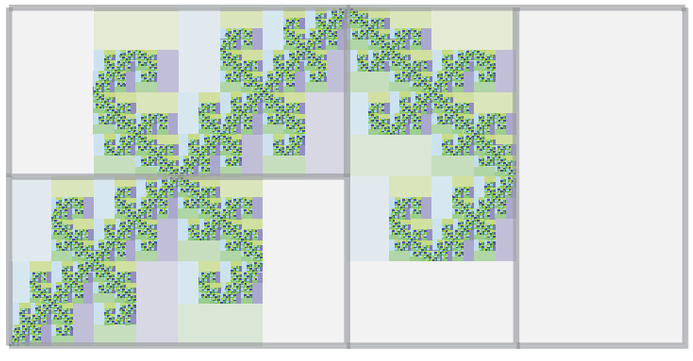

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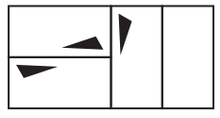

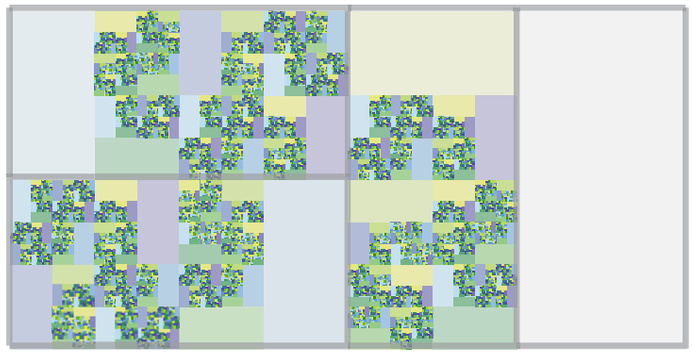

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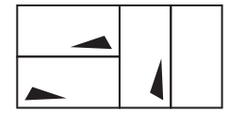

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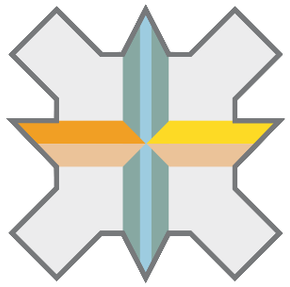
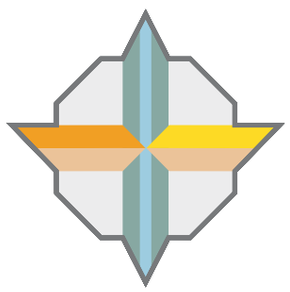
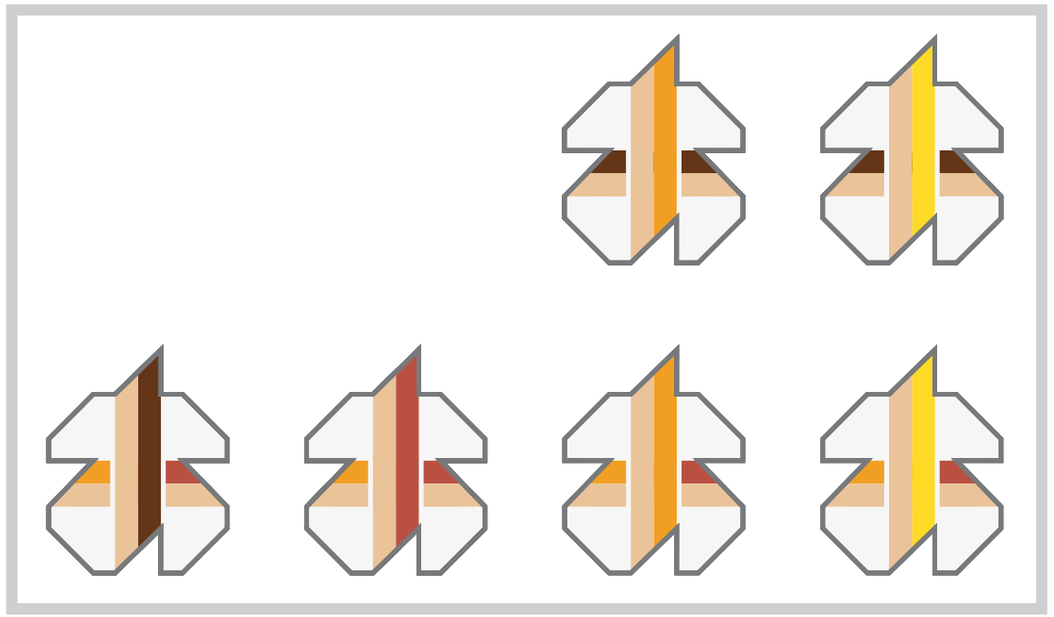


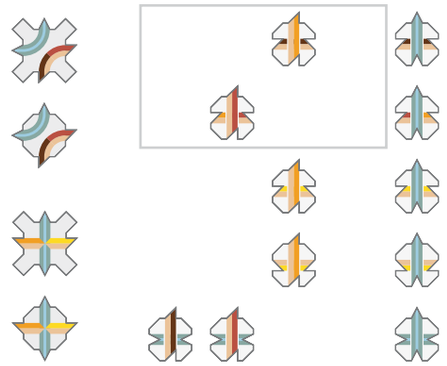




We want this informatio
they can grow transpo
tiling.

Now

Image Credit: Chaim Goodman Strauss



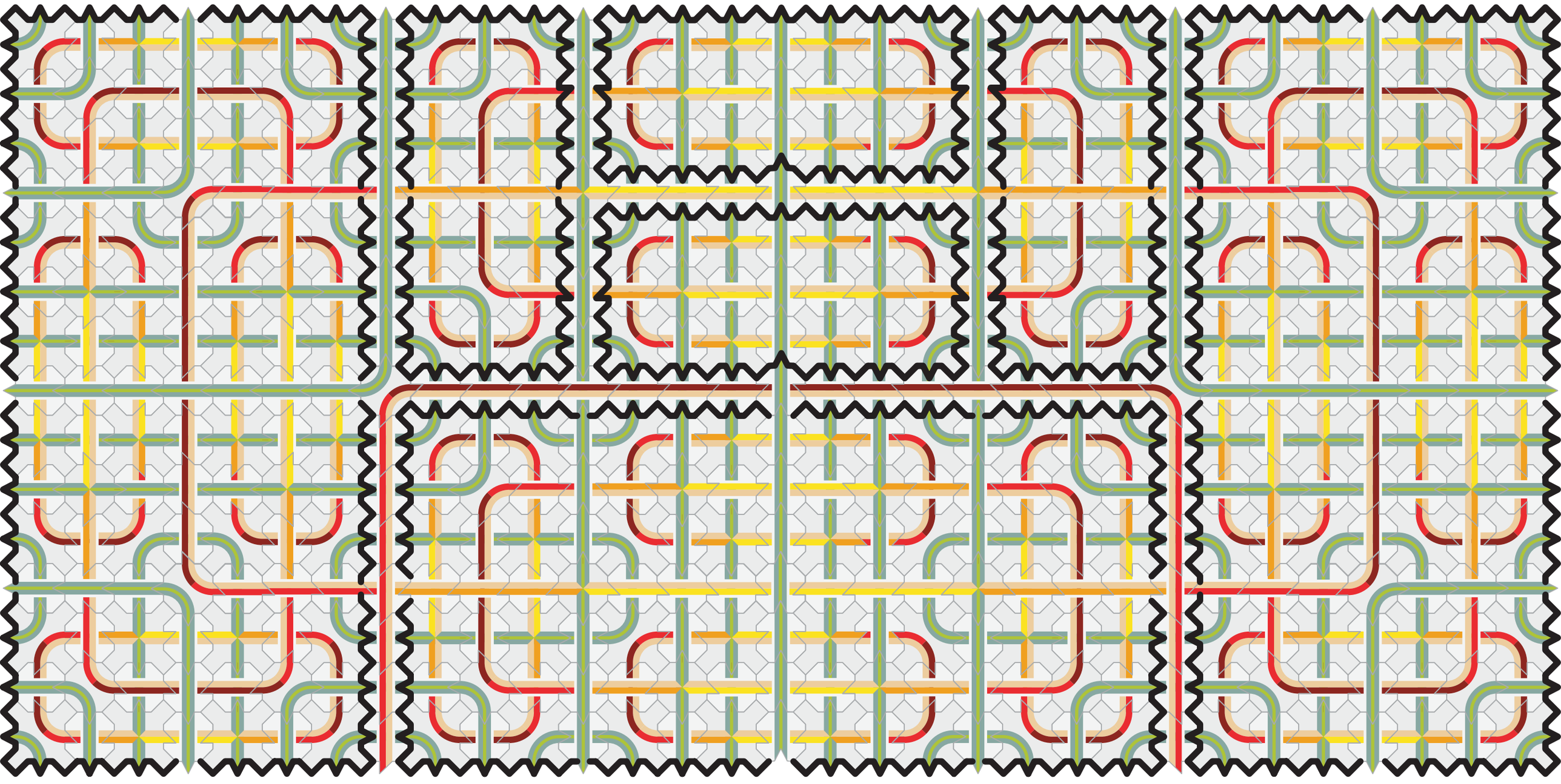


Image Credit: Chaim Goodman Strauss

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Also see lecture notes from Fernique
http://lipn.univ-paris13.fr/~fernique/qc/structure_4.pdf
https://lipn.univ-paris13.fr/~fernique/info/slides_jac.pdf

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