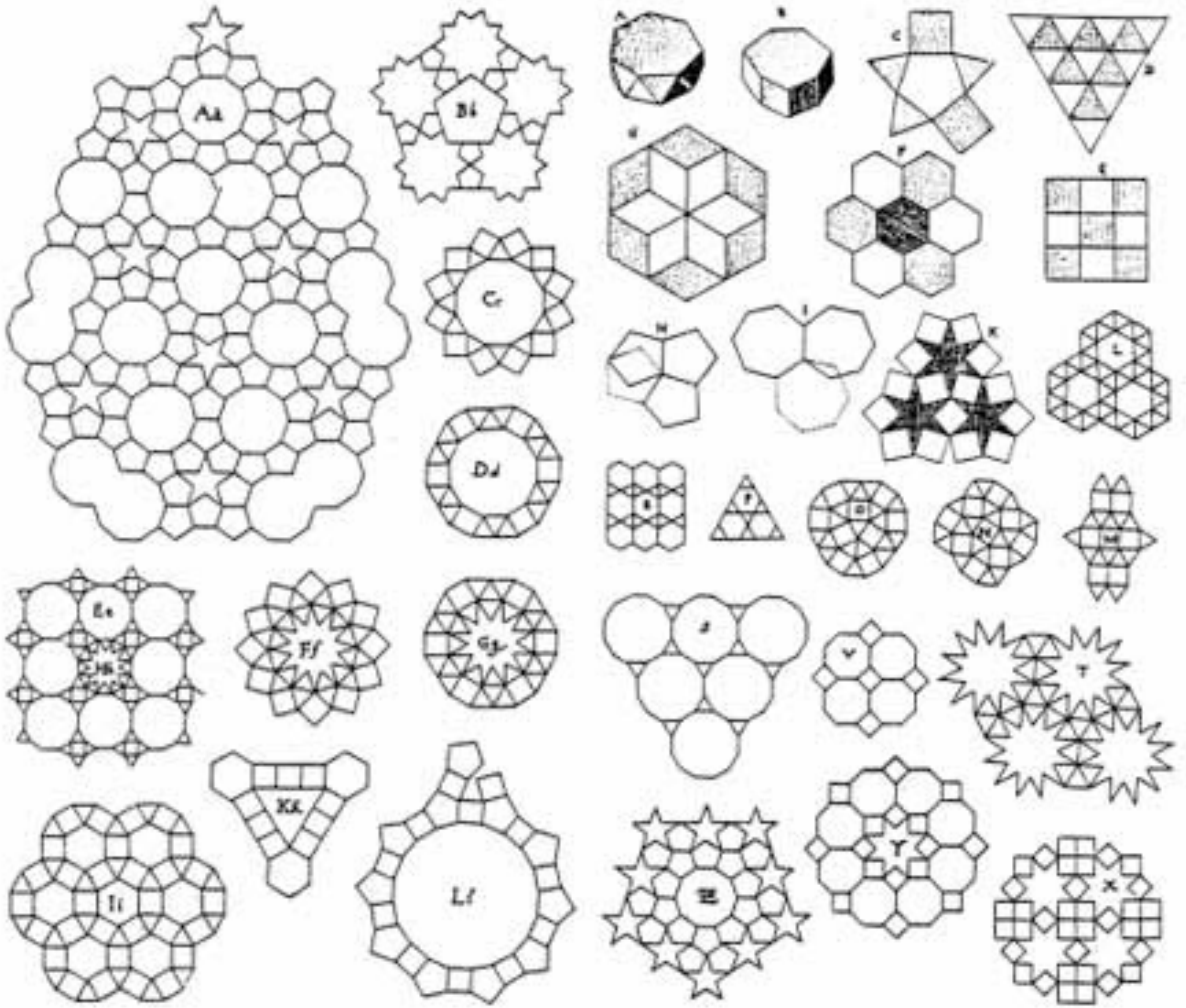
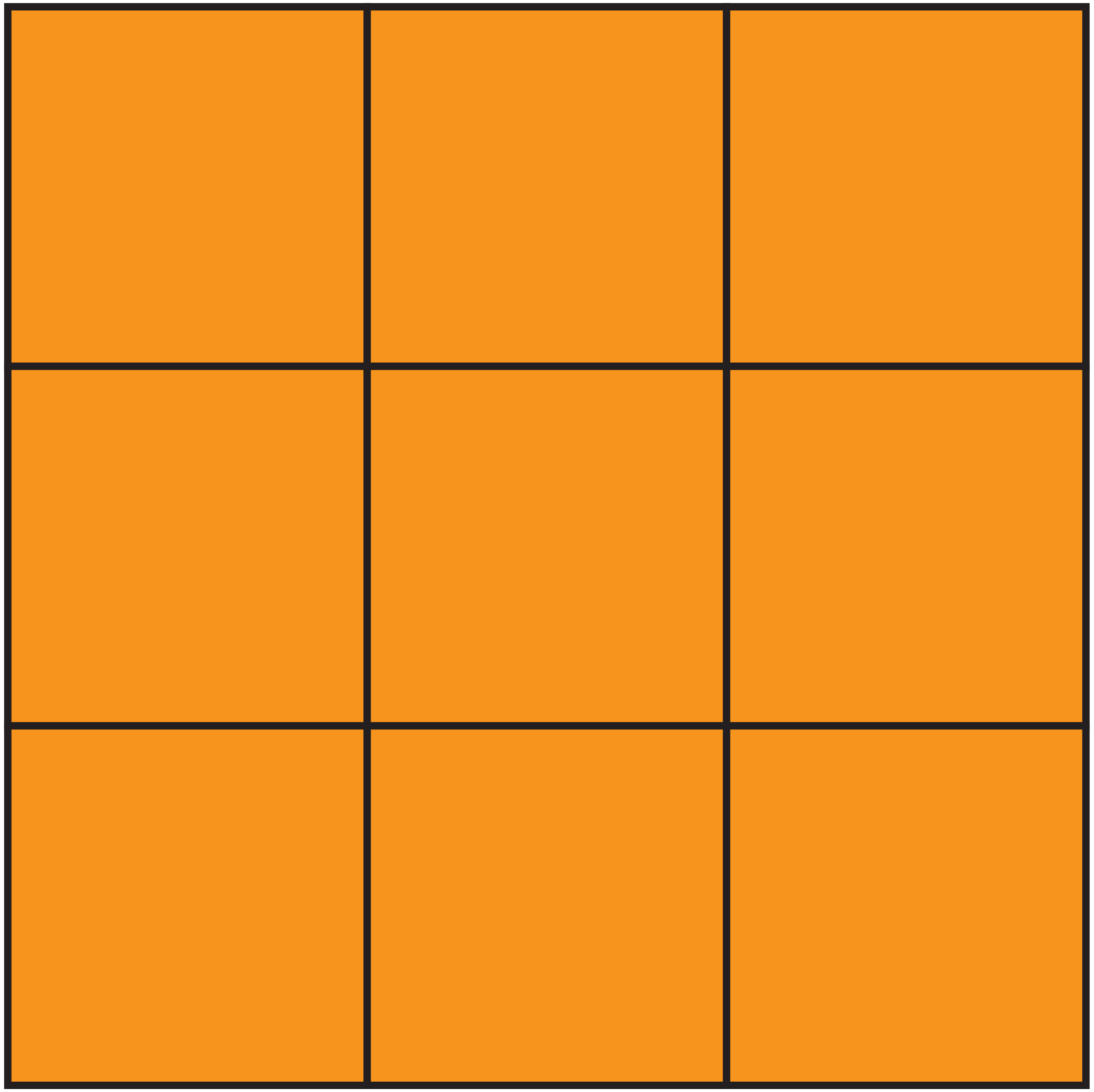
Edmund Harriss, University of Arkansas
Resident IMéRA, AMU, 2017-18
http://maxwelldemon.com
@gelada on twitter

Matching rules for Hierarchy Lecture 1



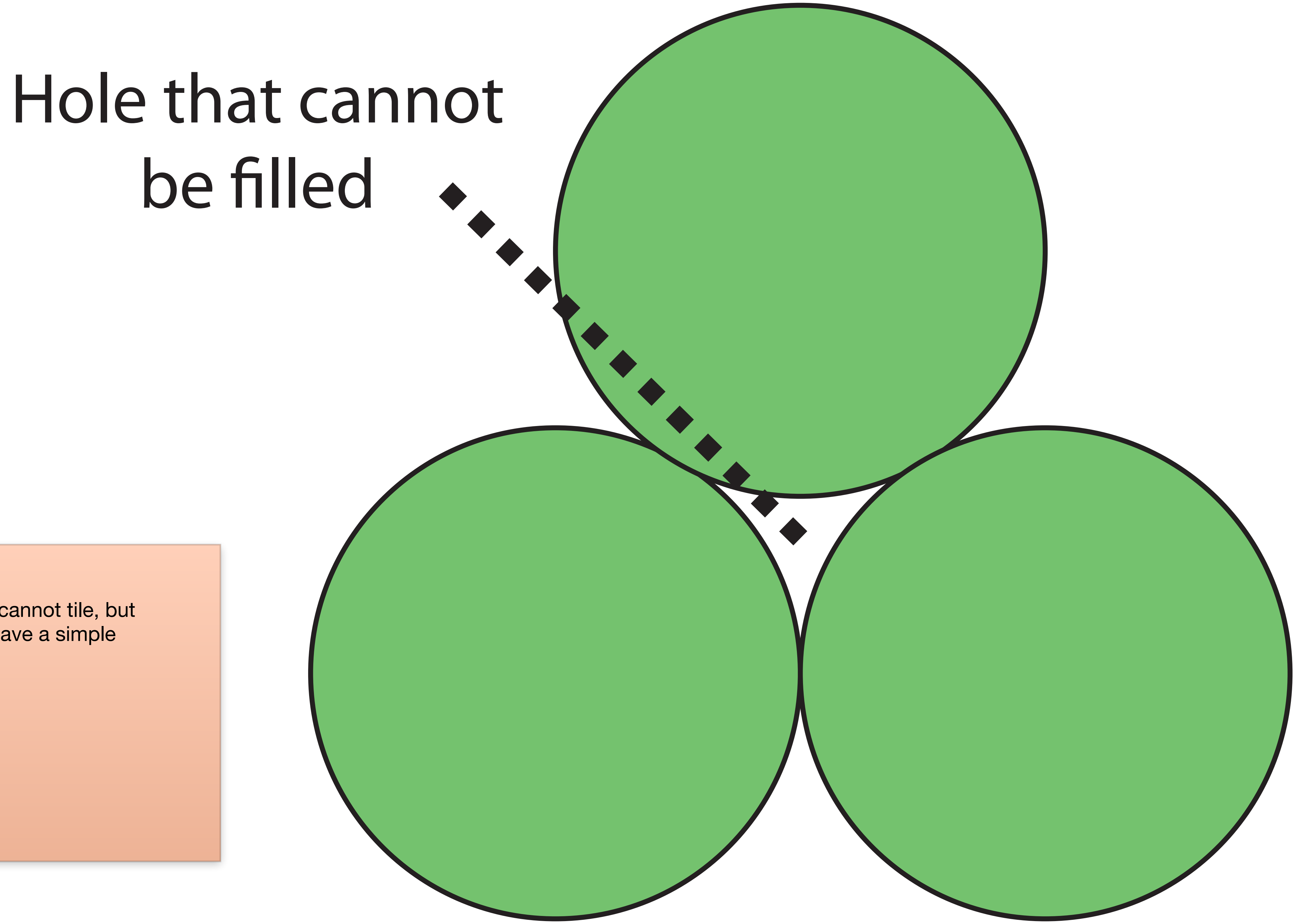
Aa
Bb
Cc
Dd
Ee
Ff
Gg
Ii
Kk
Ll



This can tile, periodically...

What about another, a circle...


BORING!

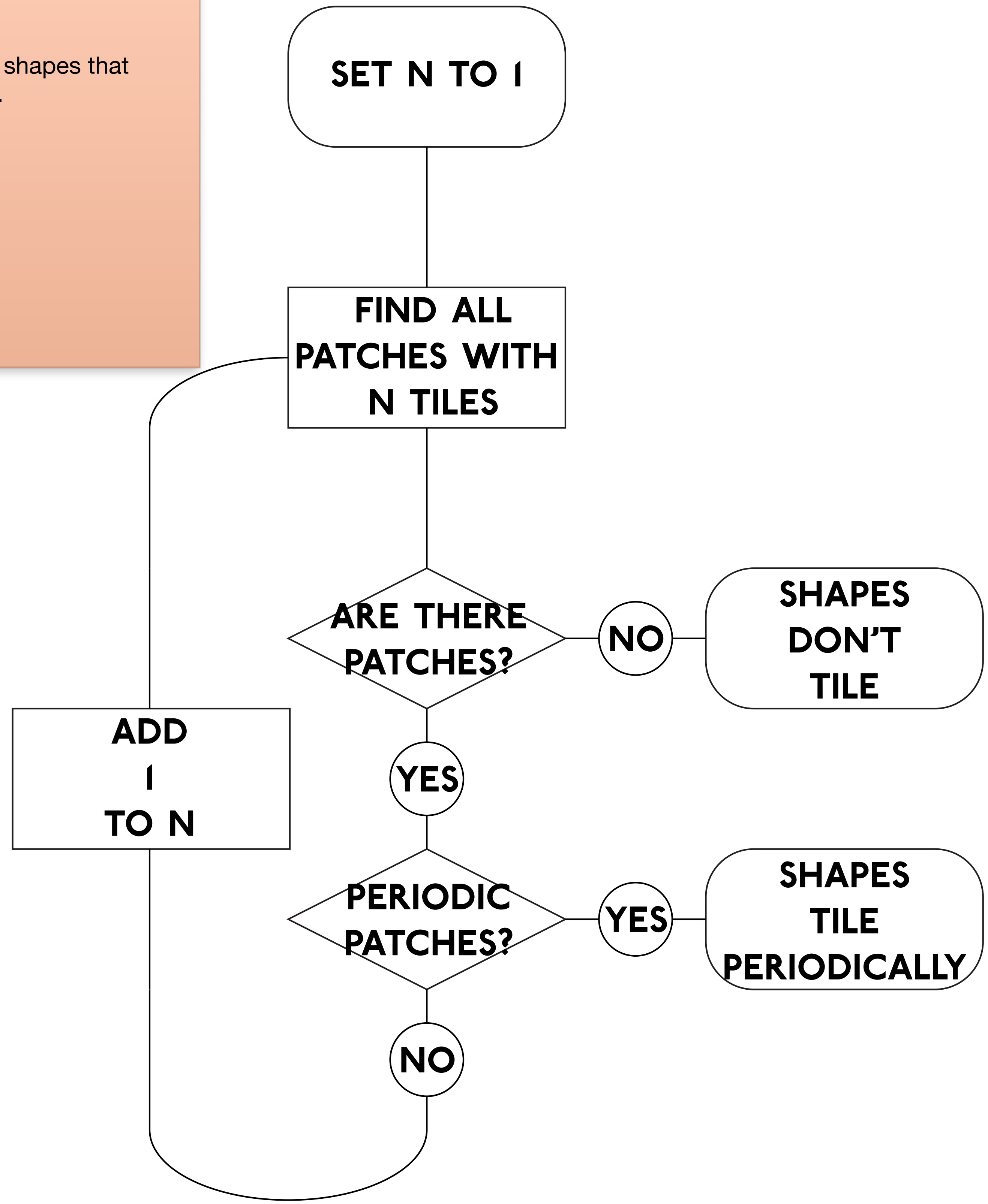


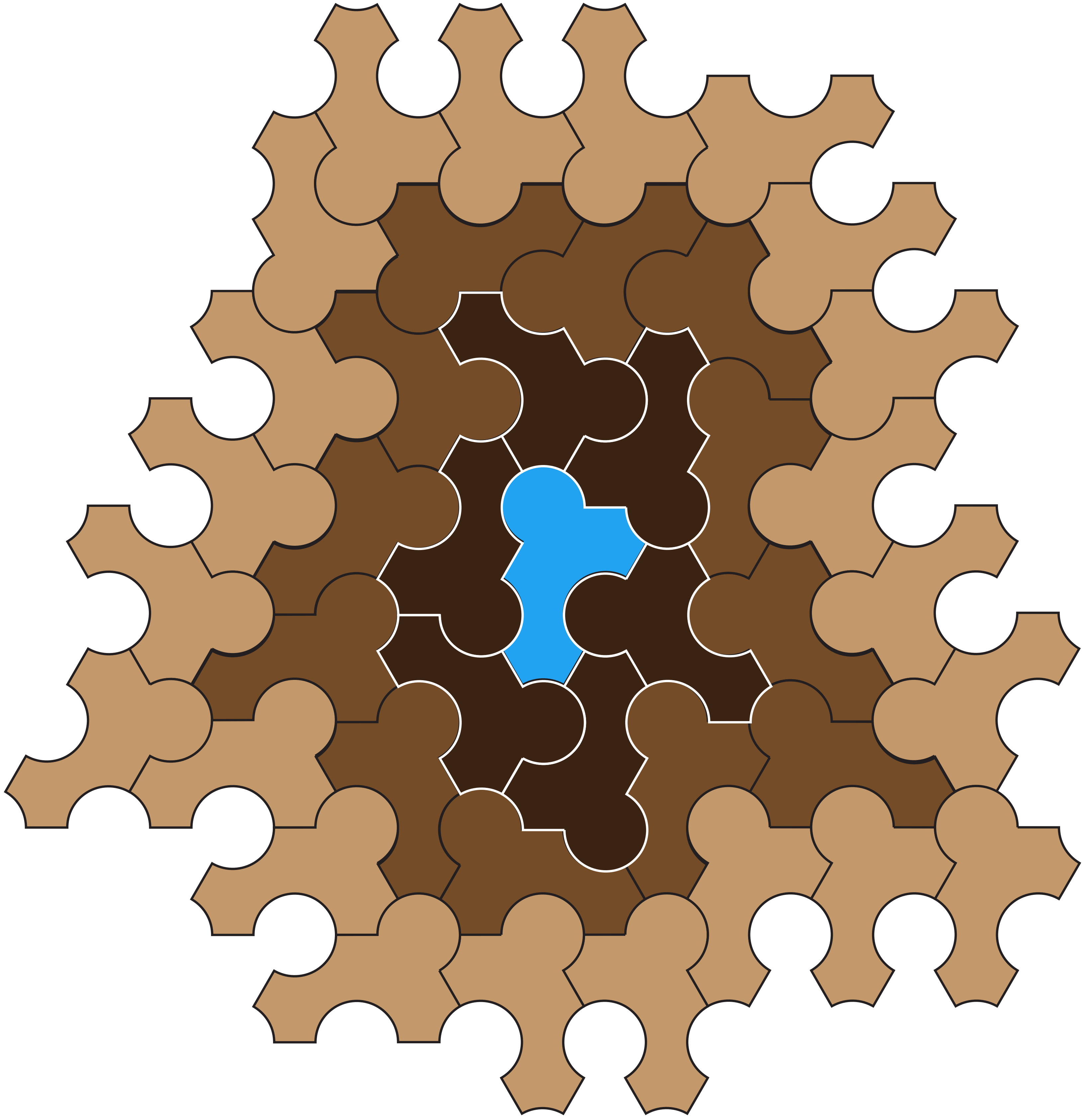
This clearly cannot tile, but maybe we have a simple algorithm...

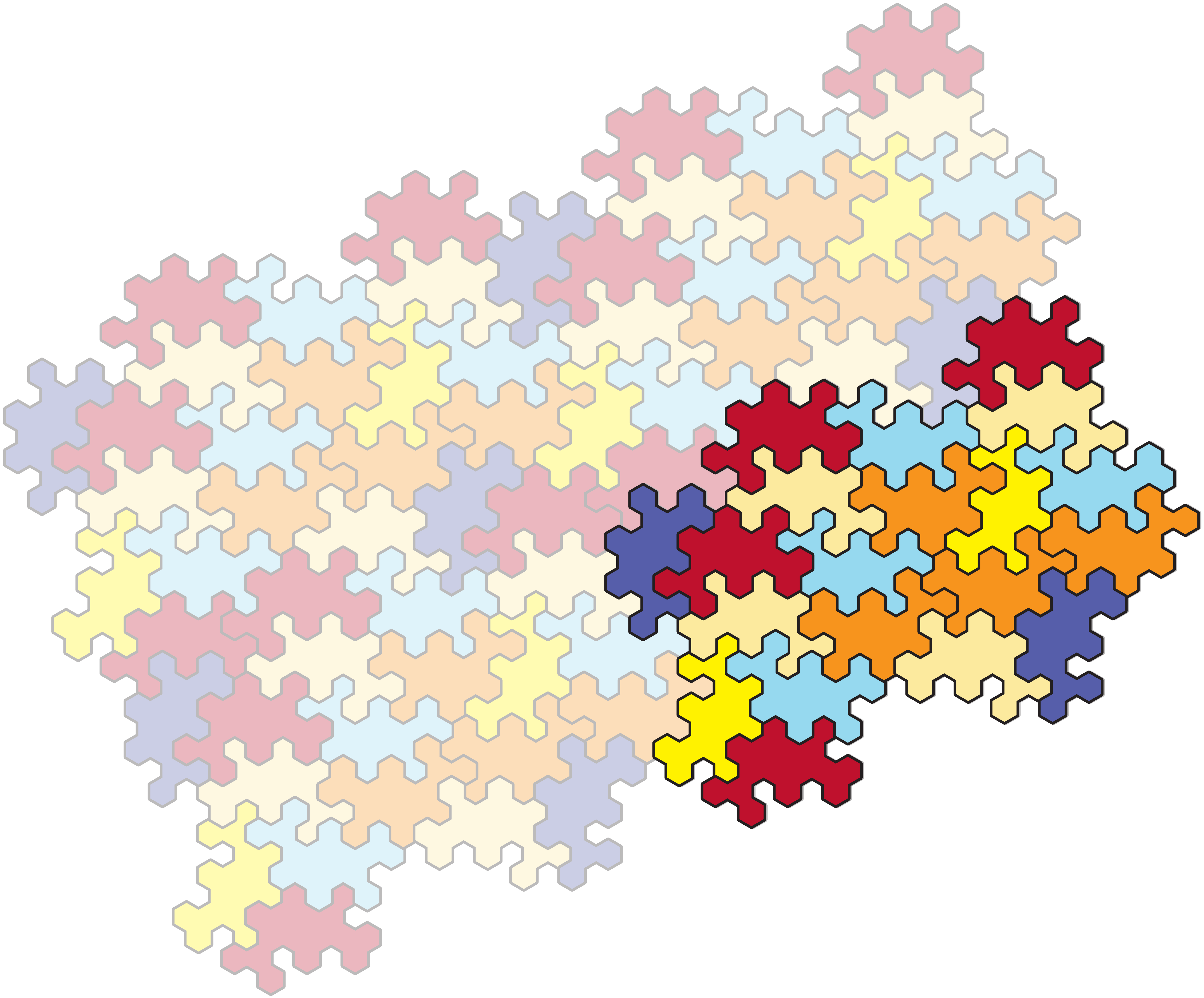
This will run around the loop forever.

It gets worse. The question is undecidable

Whatever algorithm you find, there is always a set of shapes that will cause it to run forever without giving an answer...







Berger PhD Thesis

Simpler Proof: Robinson

Current state of the art:
5 Tiles Ollinger

This is a reminder of how deep undeciability cuts into mathematics.

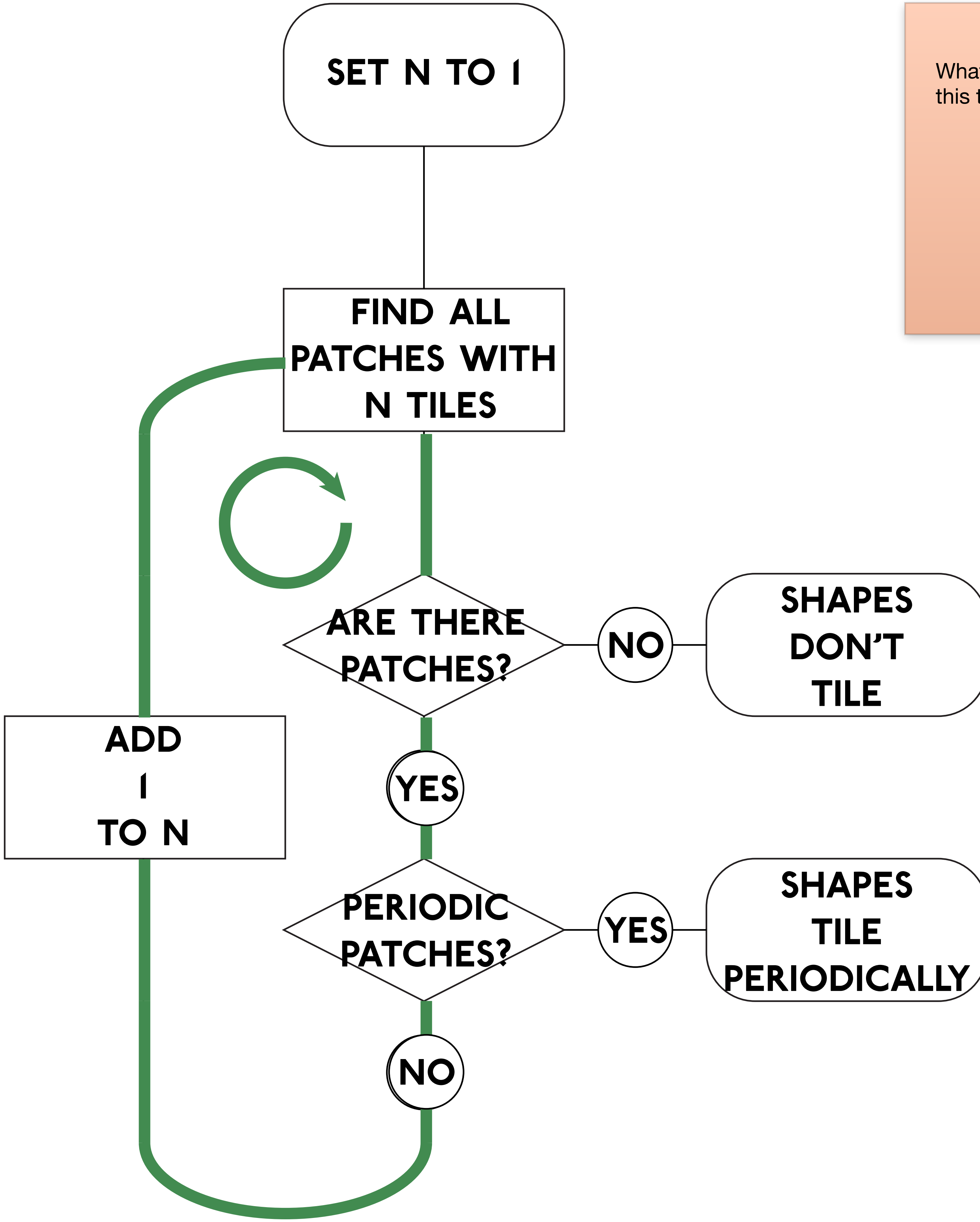
(of course it is also exciting: simple questions about tilings will always yield interesting new ideas:

Like Aperiodicity...

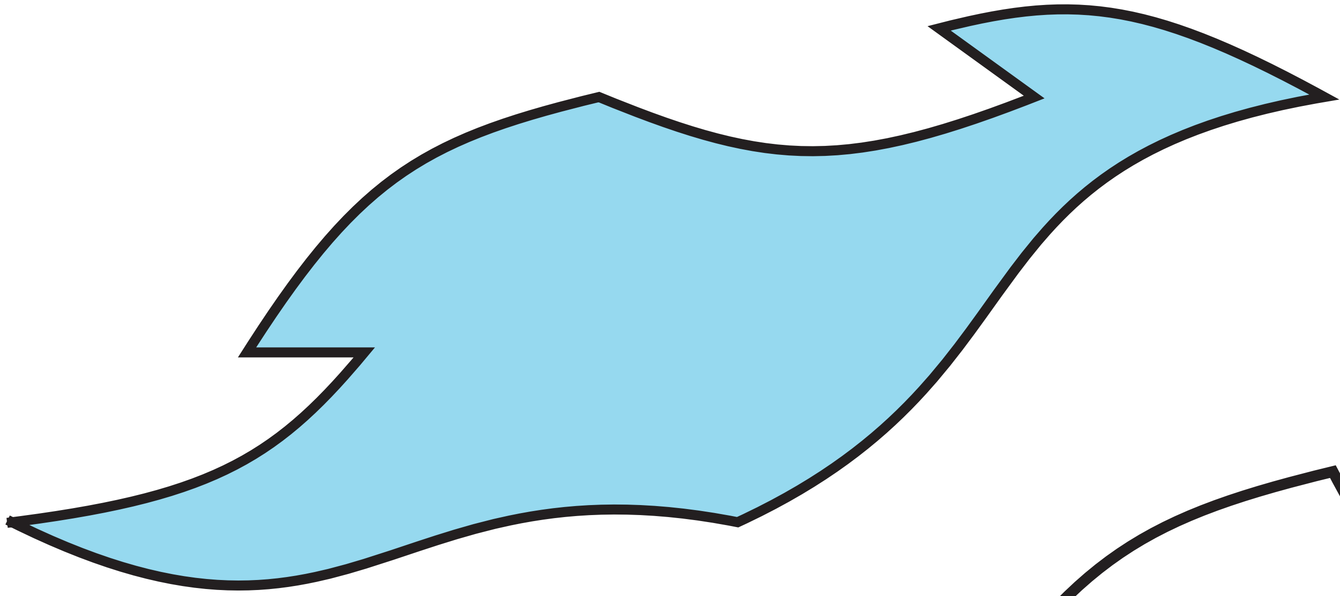
Robert Berger, *The undecidability of the domino probl*
Memoirs of the AMS 66, 1966

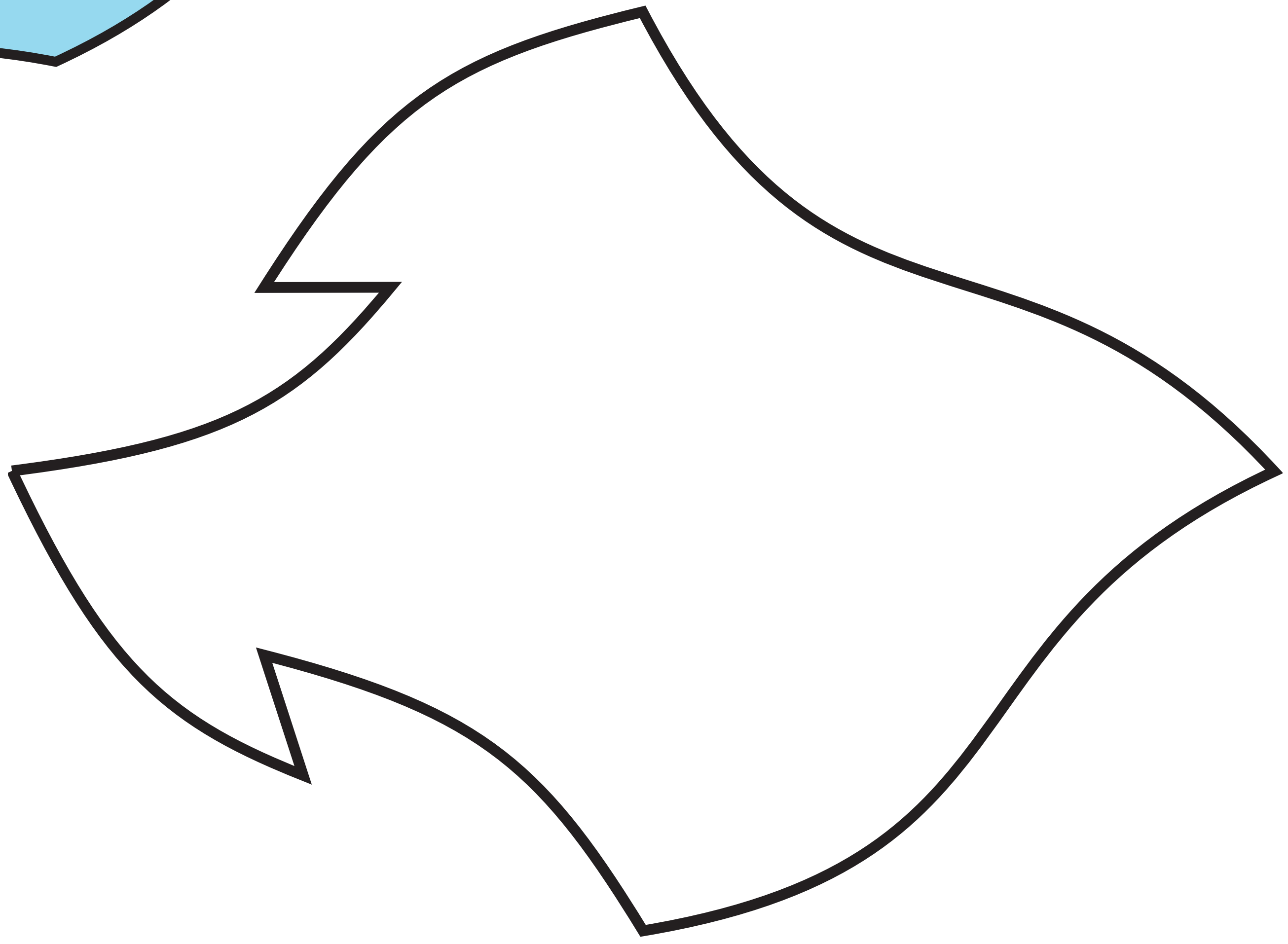
Raphael Robinson, *Undecidability and nonperiodicity fo*
Inventiones Mathematicae 12, 1971, pp. 177-209

Nicolas Ollinger: *Tiling the Plane with a Fixed Number of Polyominoes.* Proceedings of LATA 2009, Lecture Notes in Computer Science 5457, Springer 2009, pp. 638-647.



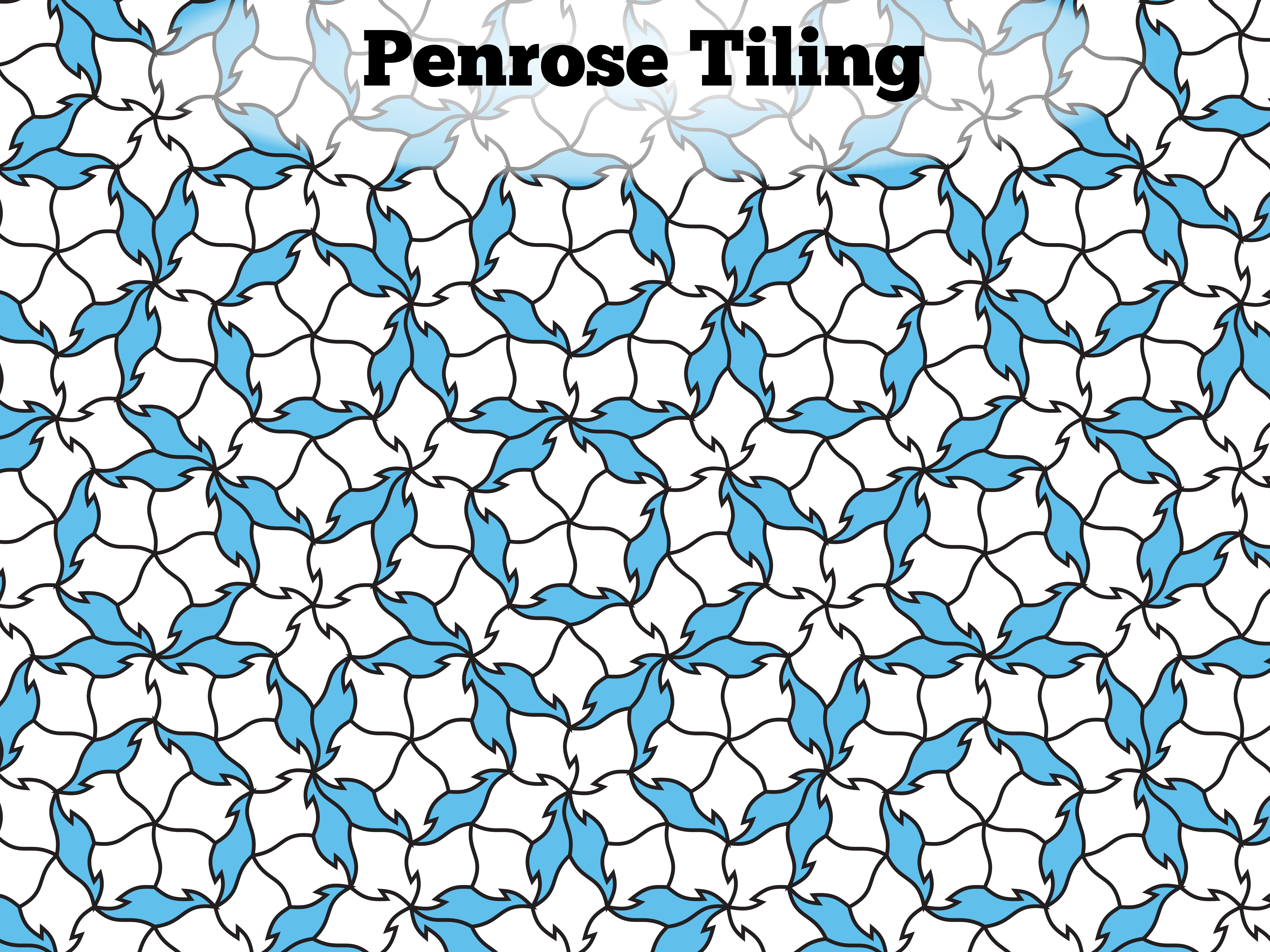
What happens when we apply this to our shapes...

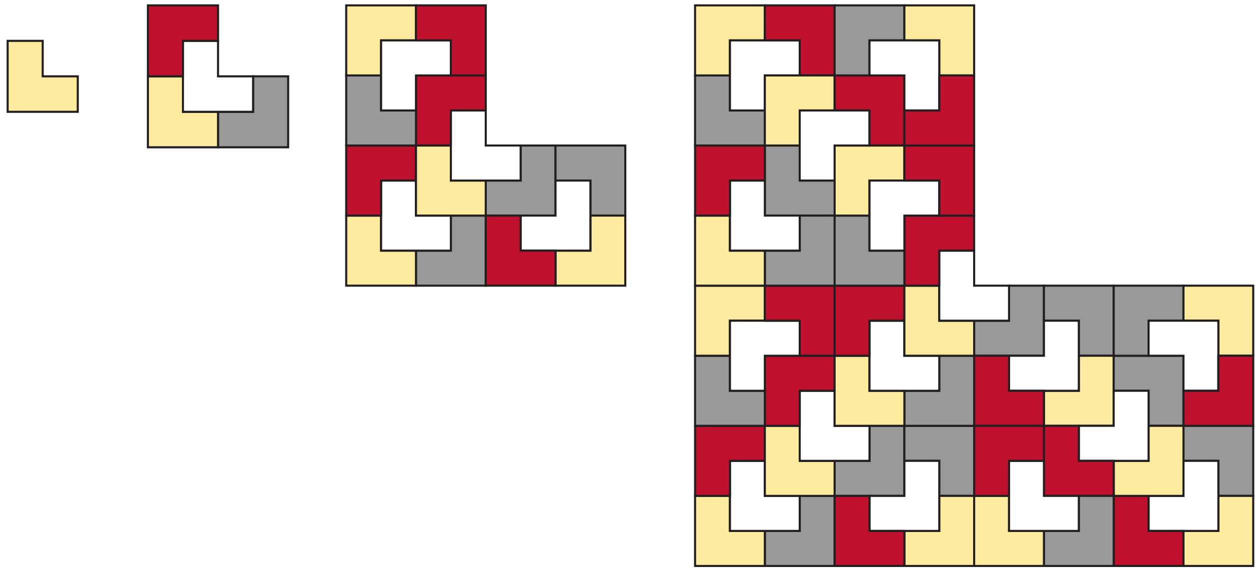


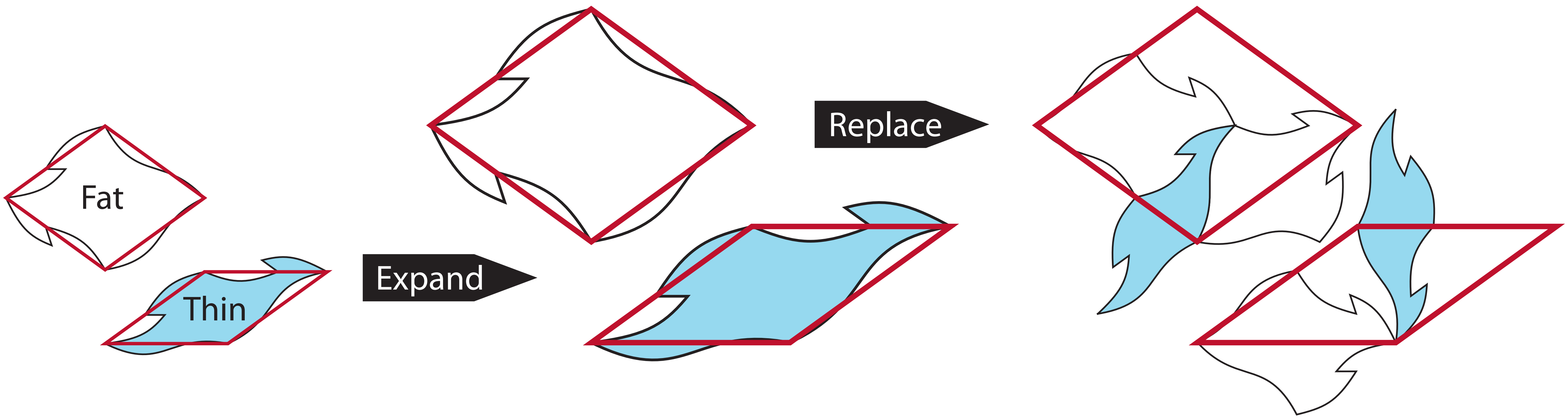


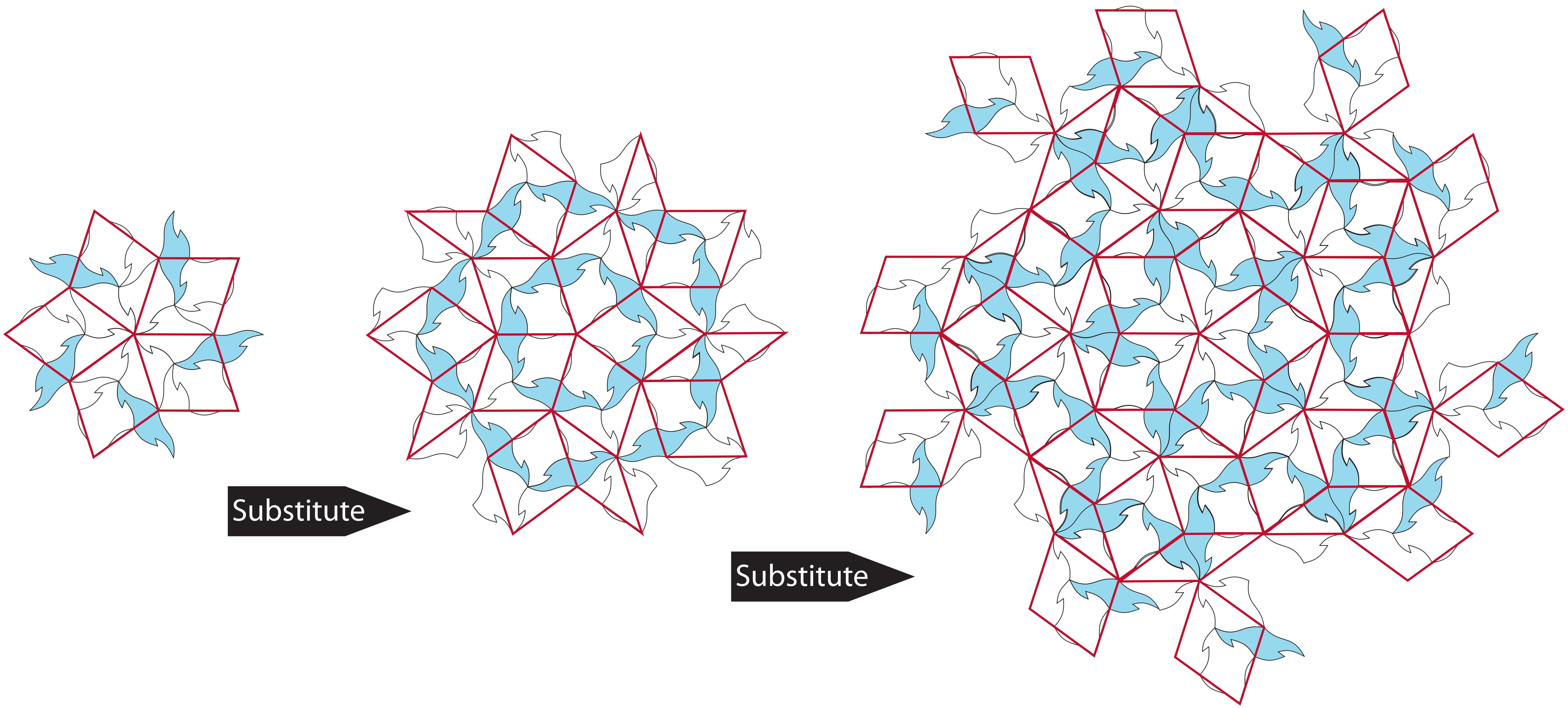
What about the third set...

Penrose Tiling






Fat
Thin
Expand
Replace


Substitute
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Definition: We will keep a naive sense of **tiling**, but the general definition is a partition of the plane by bounded regions that are the closure of their interior, and intersect only on their boundary.

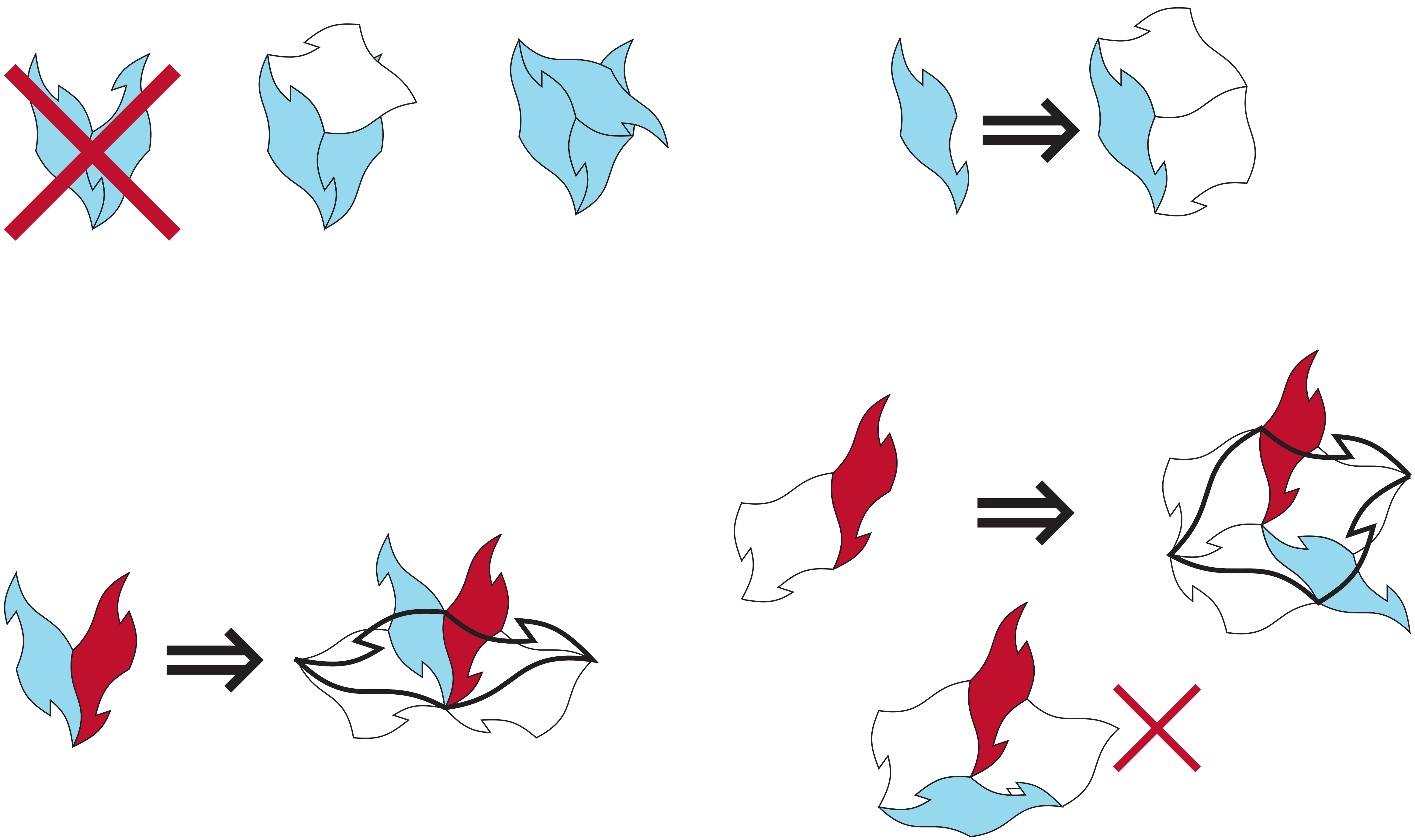
Definition: A tiling T has a **predecessor** under a substitution rule $\boldsymbol{\sigma}$ if there exists a tiling T' such that $\boldsymbol{\sigma}(T') = T$.

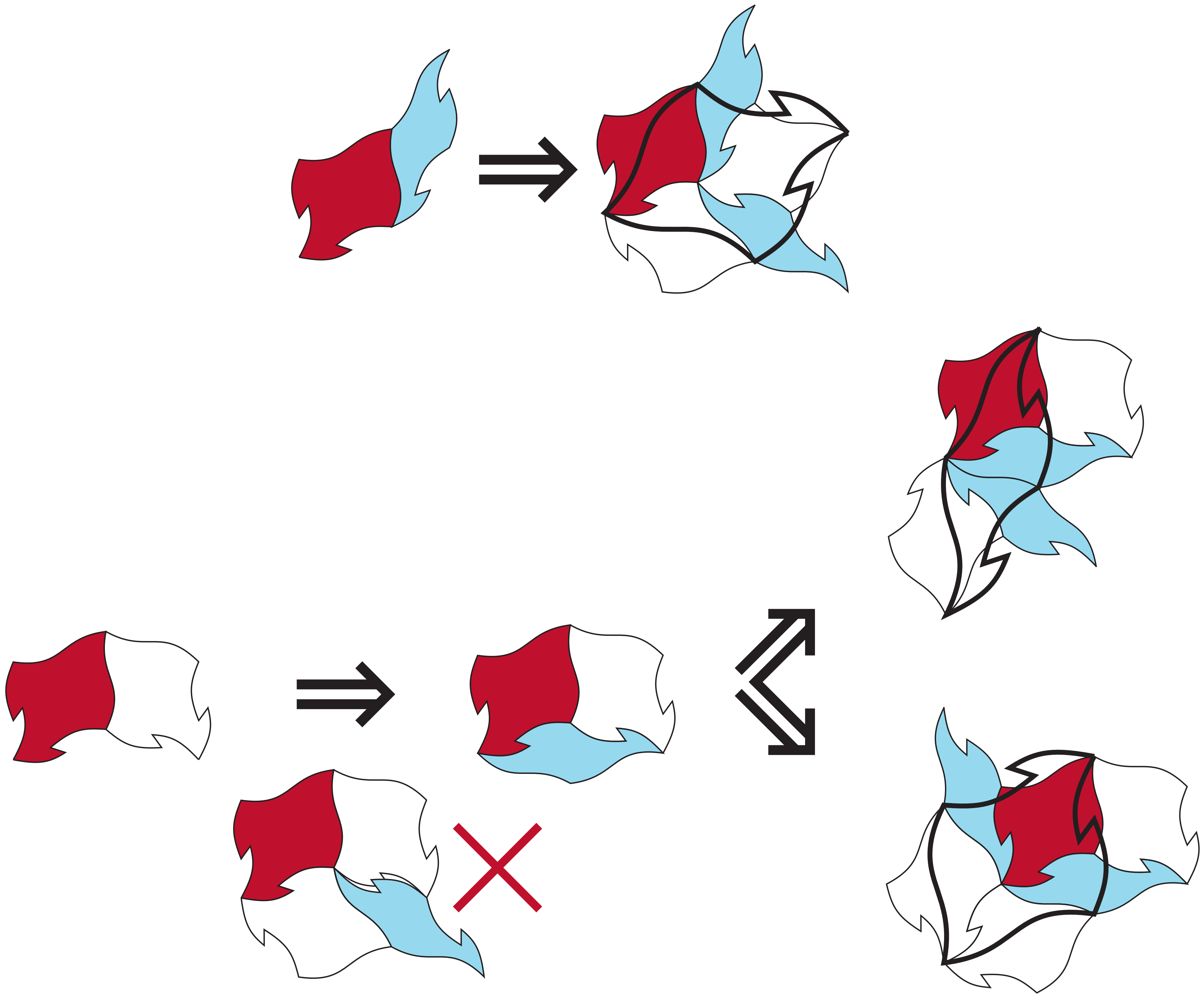
Definition: The **tilings for a substitution** rule are the tilings with an infinite string of predecessors. A substitution is **uniquely hierarchical** if each tiling has a unique predecessor.

Proposition: The Penrose tilings for the substitution rule are not periodic.

Proof 1: Ratio of tiles in the limit, found using the substitution matrix.

Proof 2: Use the hierarchy, if the tiling is uniquely hierarchical (not yet proven for Penrose) and periodic then we can use the predecessor to make the period shorter, as there must be a minimum period for a periodic tiling, this is not possible.





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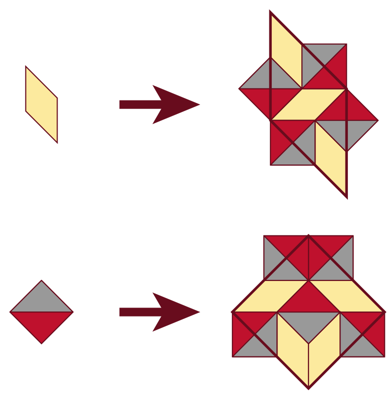
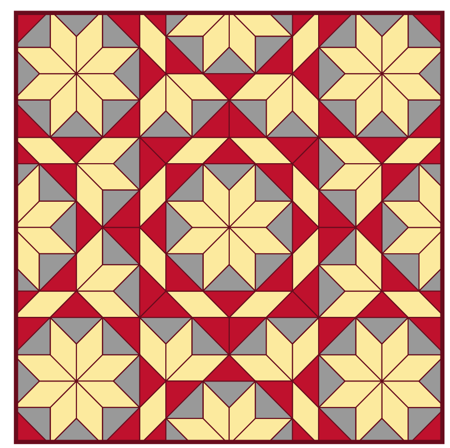
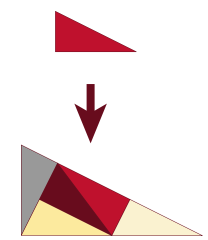
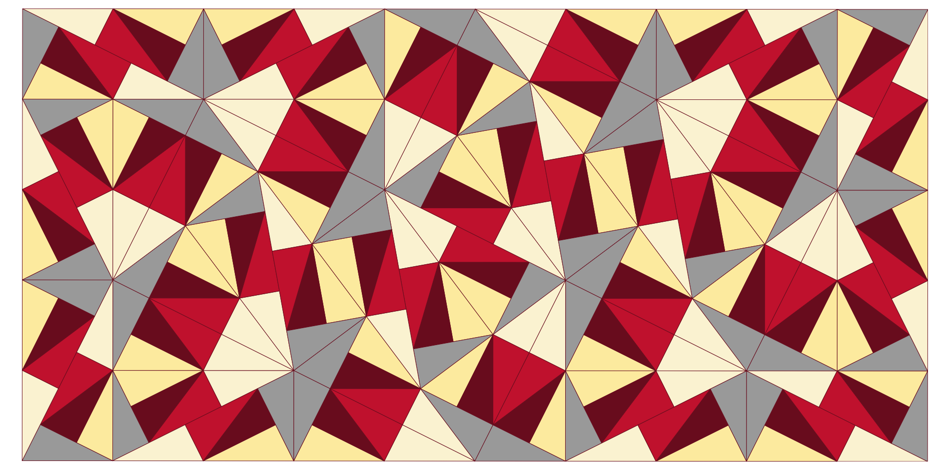
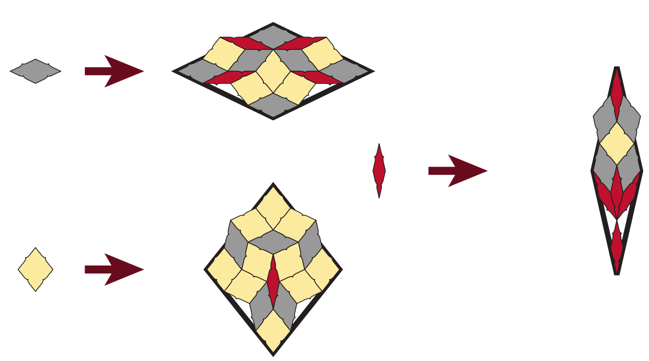
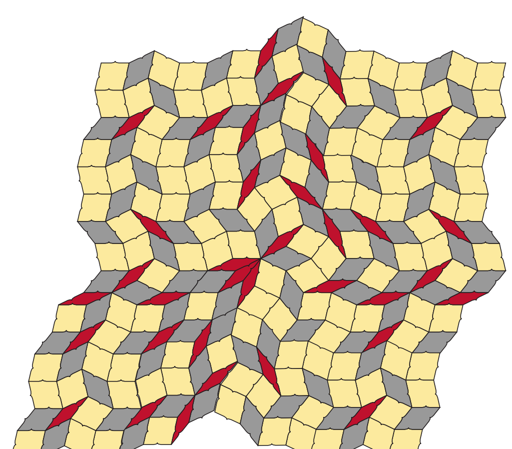


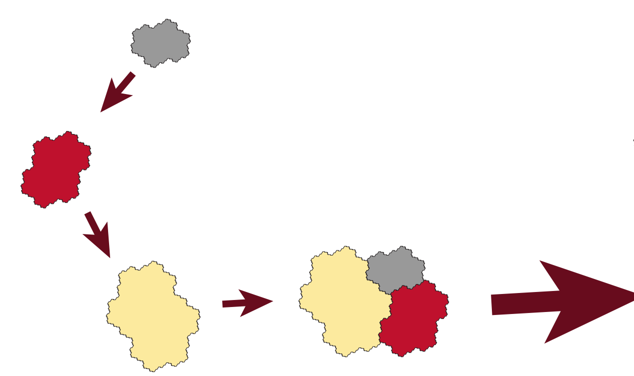


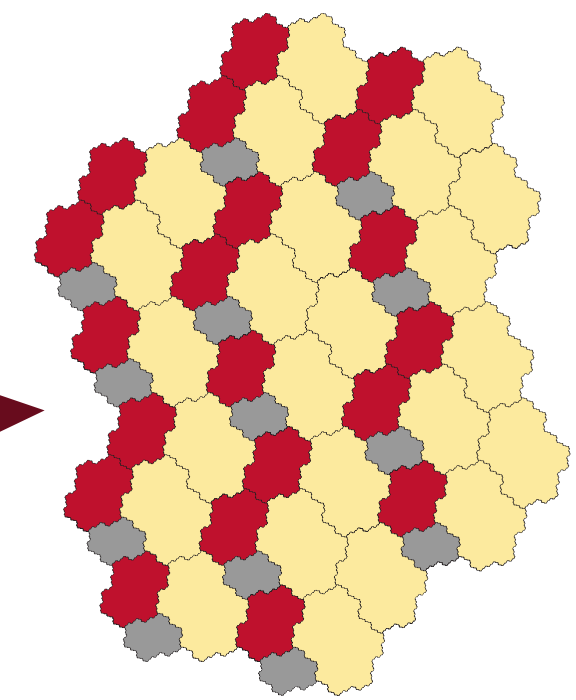












My work has been in creating and attempting to characterize substitution tilings.

The process of taking a substitution tiling

EG Penrose rhombs

and changing the edges to give a set of aperiodic tiles became known as matching rules...

A series of papers leading to a result that shows that we can get an aperiodic set of shapes from any substitution tiling...

Q: How?

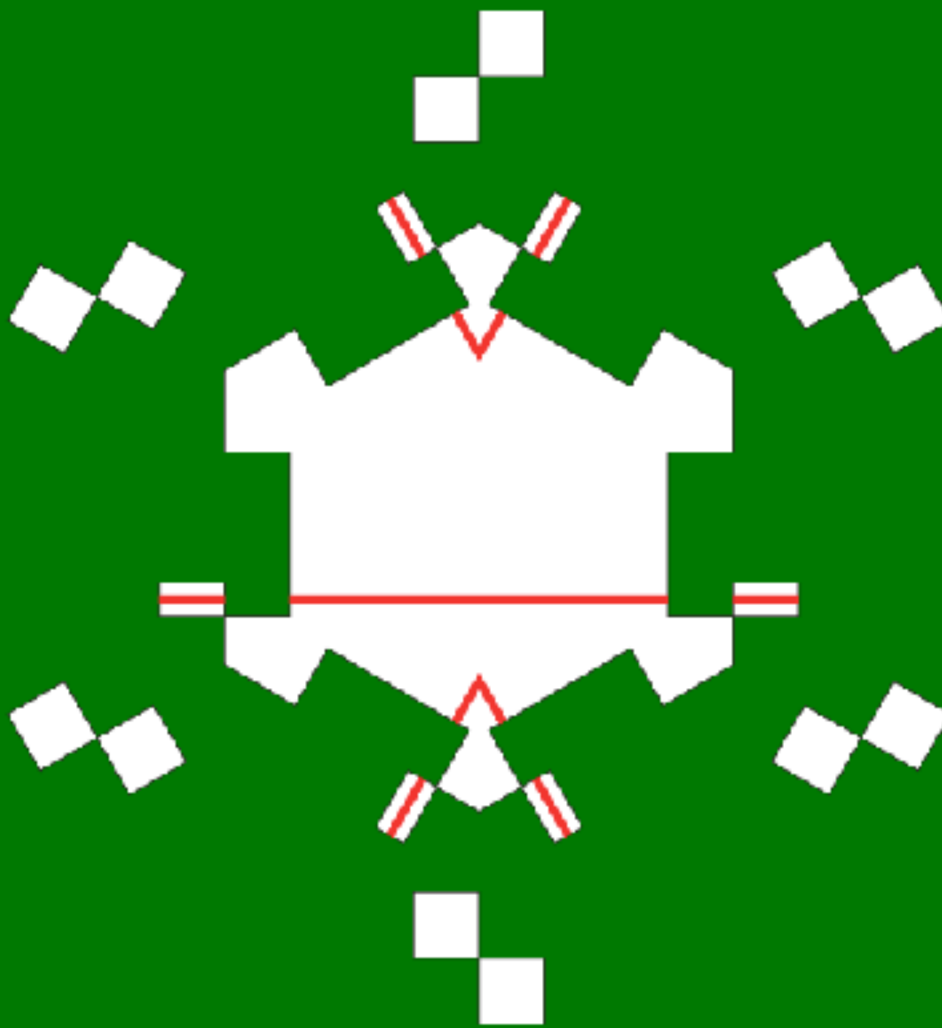
This is an important result, but not well understood. So now...

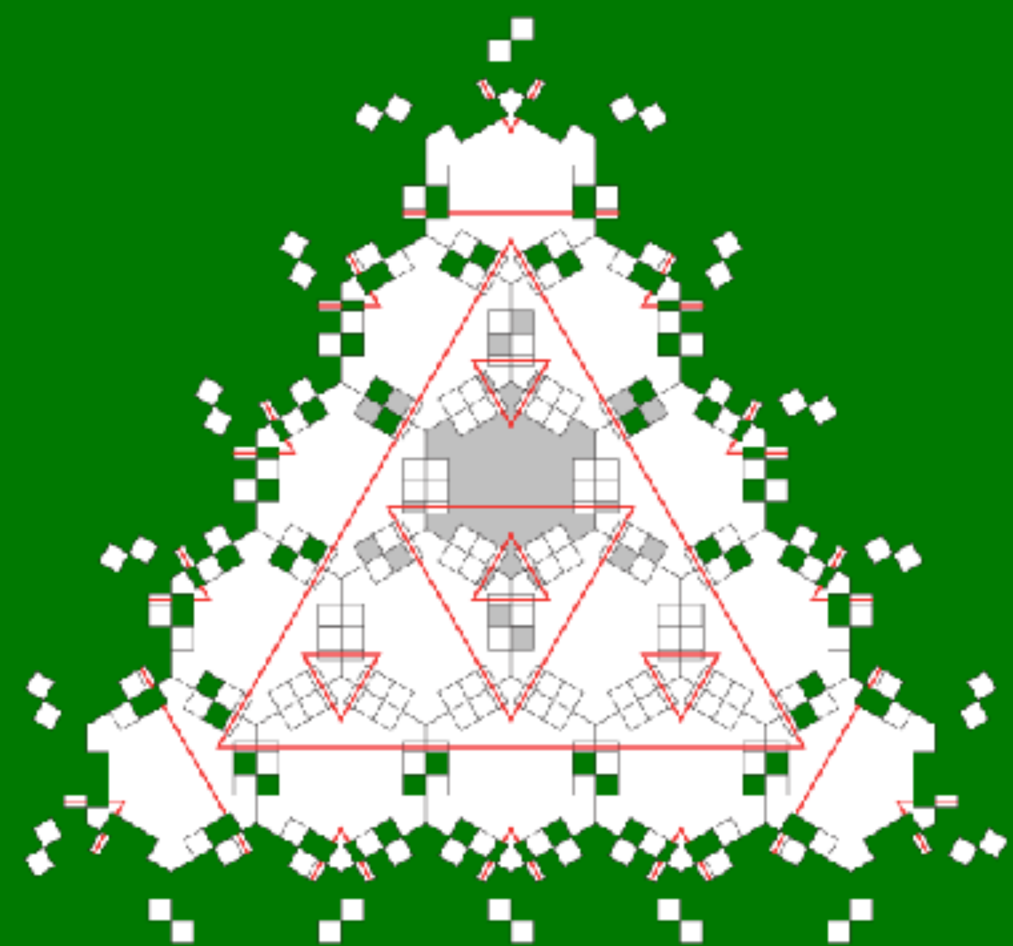
Raphael Robinson, *Undecidability and nonpe*
Inventiones Mathematicae 12, 1971, pp. 177-

Sharhar Mozes, *Tilings, substitution system*
them, J. D'Analyse Math. 53, 1989, pp.139-186

Chaim Goodman-Strauss, *Matching rules and substitution tilings,*
Annals of Mathematics 147 No. 1, 1998, pp. 181-223

Thomas Fernique, Nicolas Ollinger, *Combinatorial Substitutions and Sofic Tilings,*
TUCS, Journées Automates Cellulaires 2010, Dec 2010, Turku, Finland. pp.100-110





Joshua Socolar and Joan Taylor,
An aperiodic hexagonal tile,
Journal of Combinatorial Theory Series A
Volume 118 Issue 8, November, 2011
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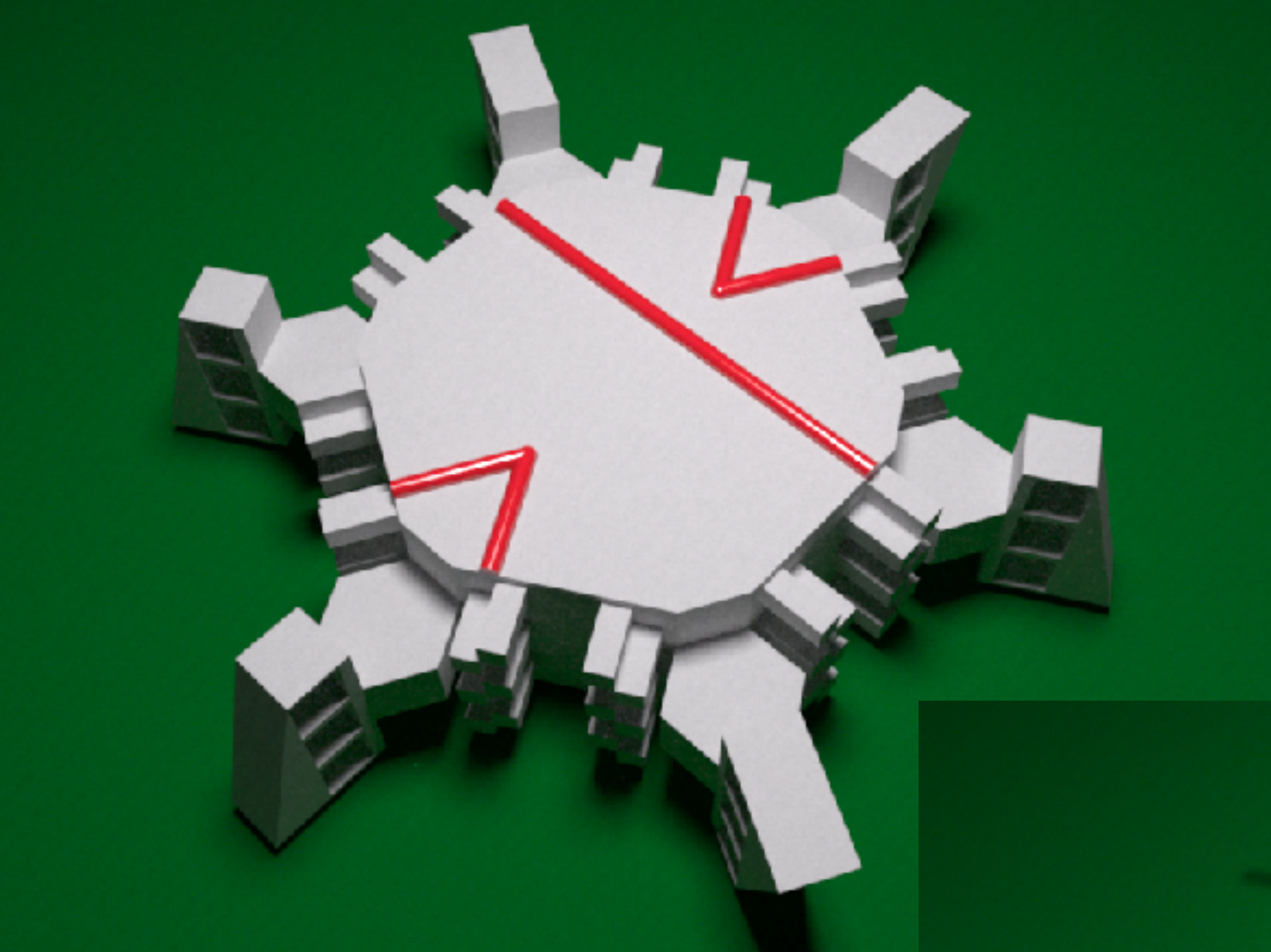
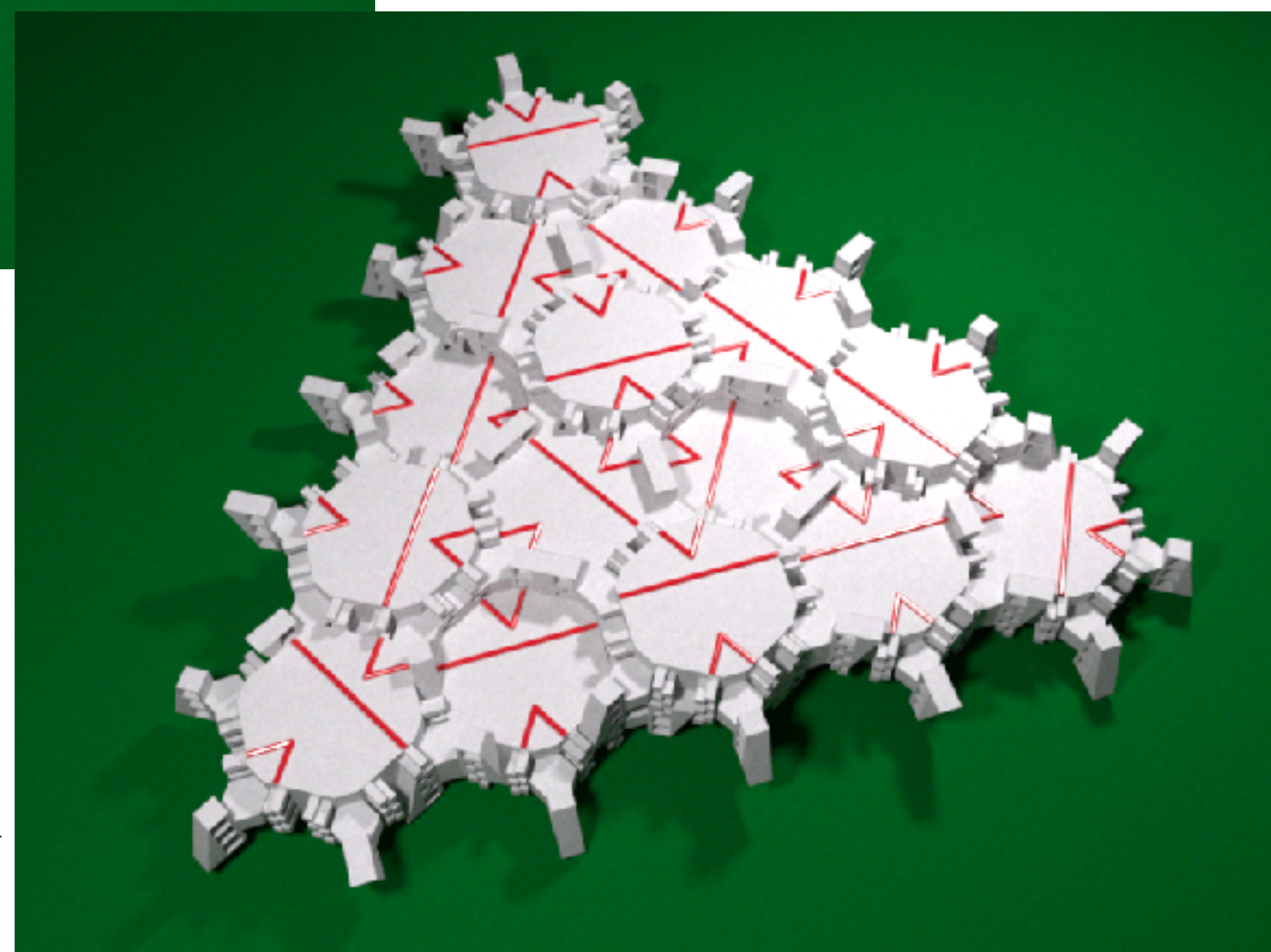


3d version, 1 periodic direction

Mentioned in New Scientist

How can you tell if these shapes tile at all?

The answer is a substitution rule.



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