Homology groups of generalized polyomino type tilings

Research school on Aperiodicity and Hierarchical structures in tilings, 18 - 22 September 2017
Lyon, France

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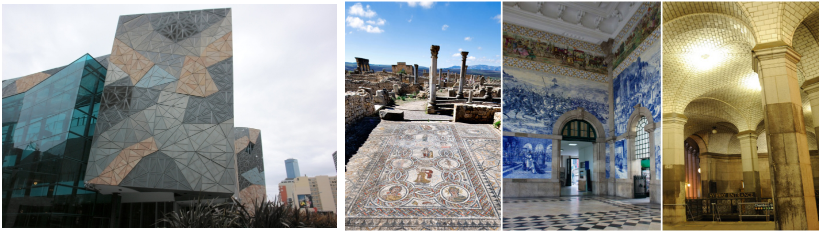


Figure: Tilings in atrs and poplular coluture

Tiling problems

Tiling problems

- Region for tiling

Tiling problems

- Region for tiling

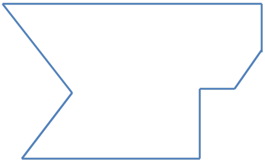
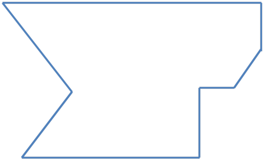


Figure: M_1=Finite region

Tiling problems

- Region for tiling



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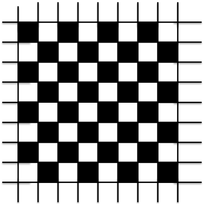
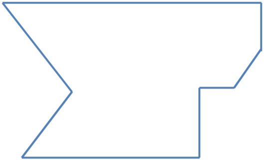


Figure: M_2 = Plane

Tiling problems

- Region for tiling



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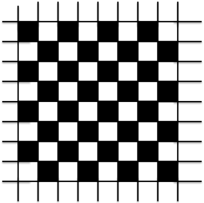

Figure: M_2 = Plane

Figure: M_3 = Surface

- A finite set Σ of tiles

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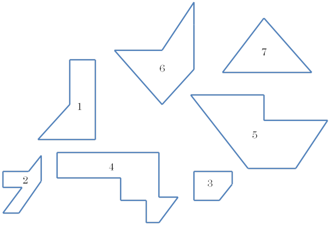
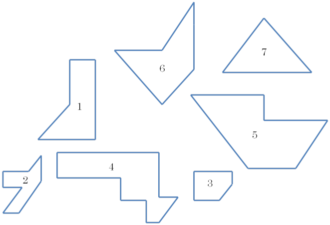


Figure: Σ_1

- A finite set Σ of tiles



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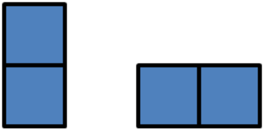
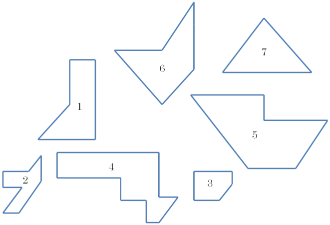


Figure: Σ_2

- A finite set Σ of tiles



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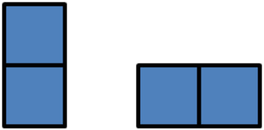


Figure: Σ_2

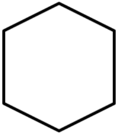
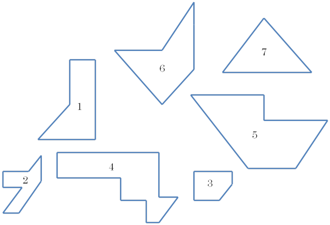


Figure: Σ_3

- A finite set Σ of tiles



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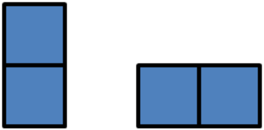


Figure: Σ_2

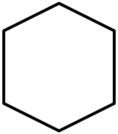
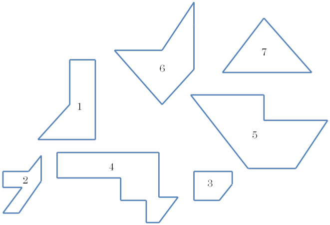


Figure: Σ_3

- Is there a tiling?

- A finite set Σ of tiles



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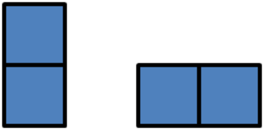


Figure: Σ_2

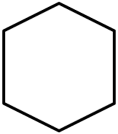


Figure: Σ_3

- Is there a tiling? How many different tilings are there?

- A finite set Σ of tiles

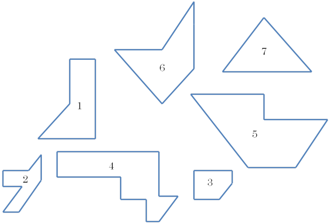


Figure: Σ_1

Figure: Σ_2

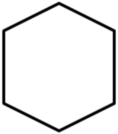
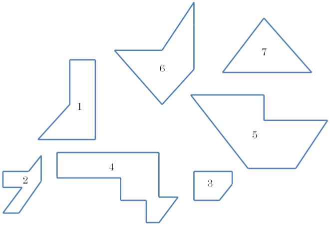


Figure: Σ_3

- Is there a tiling? How many different tilings are there?
- Is a tiling easy to find?

- A finite set Σ of tiles



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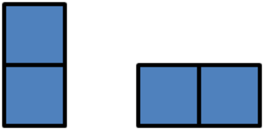


Figure: Σ_2

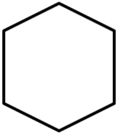
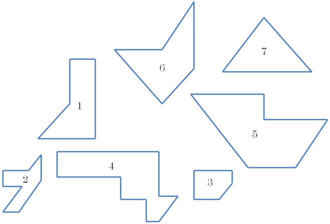


Figure: Σ_3

- Is there a tiling? How many different tilings are there?
- Is a tiling easy to find? Is it easy to prove a tiling doesn't exist?

- A finite set Σ of tiles



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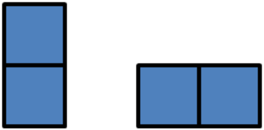


Figure: Σ_2

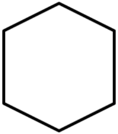


Figure: Σ_3

- Is there a tiling? How many different tilings are there?
- Is a tiling easy to find? Is it easy to prove a tiling doesn't exist?
- Is it easy to convince someone that a tiling doesn't exist?

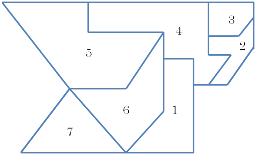
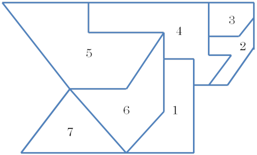


Figure: M_1



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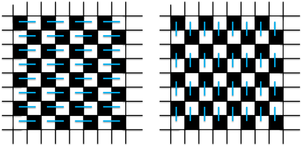
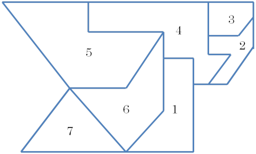


Figure: M_2



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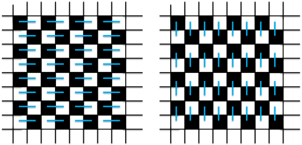

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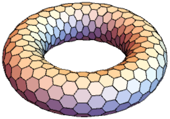

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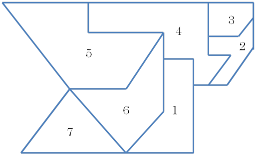
Figure: Some possible answers for given problems



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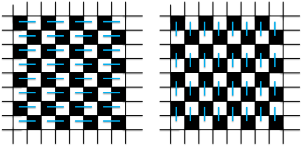
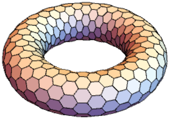


Figure: M_2



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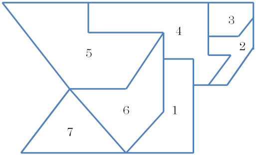

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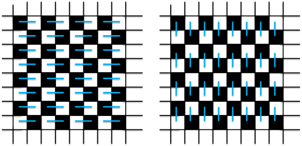


Figure: M_2

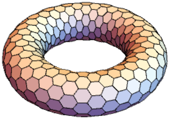
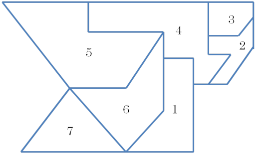


Figure: M_3

Figure: Some possible answers for given problems

- Tiling problem
  - A region M and finite set Σ of tile



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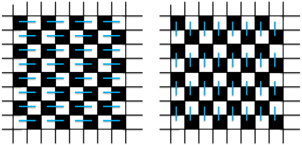


Figure: M_2

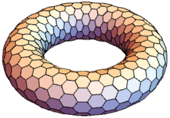


Figure: M_3

Figure: Some possible answers for given problems

- Tiling problem
  - A region M and finite set Σ of tile
  - Does Σ tile the M?

- Shapes

- Shapes

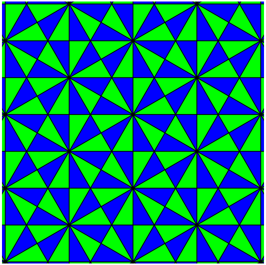
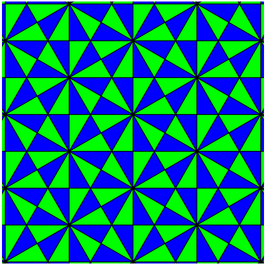


Figure: Triangular lattice (Polyamonds)

- Shapes



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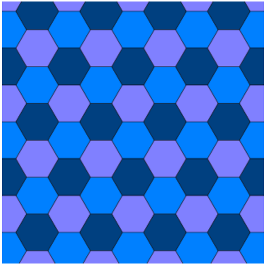
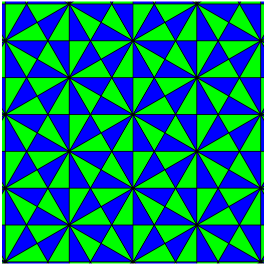


Figure: Hexagonal lattice (Polyhes)

- Shapes



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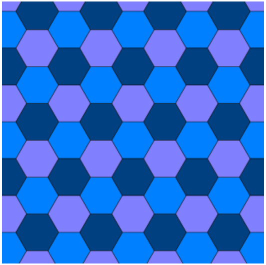

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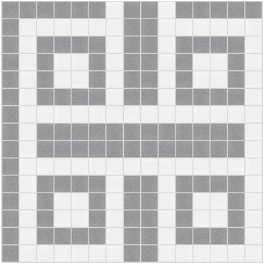
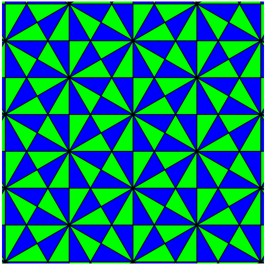


Figure: Square lattice (Polyominoes)

- Shapes



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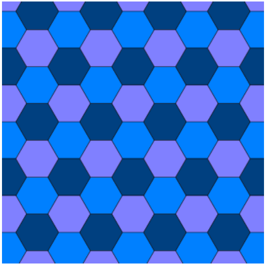

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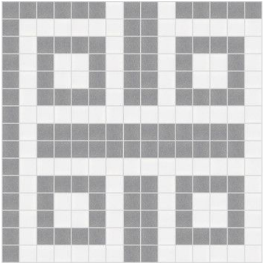

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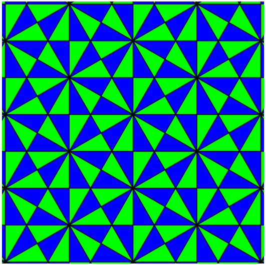
- periodic

- Shapes



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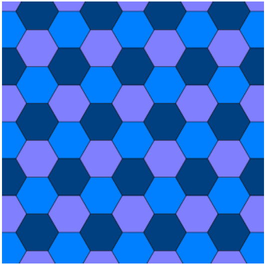

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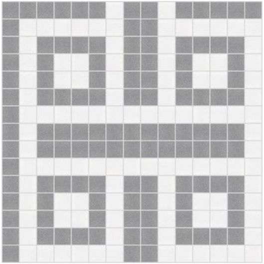

Figure: Square lattice (Polyominoes)

- periodic
- aperiodic

Polyomino

Polyomino

- Polyomino

Polyomino

- Polyomino

Figure: Polyomino

Polyomino

- Polyomino

Figure: Polyomino

Figure: Not a polyomino

Polyomino

- Polyomino

Figure: Polyomino

Figure: Not a polyomino

- Solomon W. Golomb (1965.)

Polyomino

- Polyomino

Figure: Polyomino

Figure: Not a polyomino

- Solomon W. Golomb (1965.)
- Martin Gardner Scientific American, "Mathematical Games"

Classification of polyominoes

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Figure: Monomino

Classification of polyominoes

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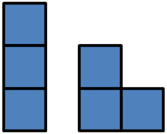
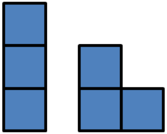


Figure: Trominoes

Figure: Domino

Classification of polyominoes

Figure: Monomino



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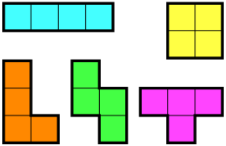


Figure: Tetrominoes

Classification of polyominoes

Figure: Monomino

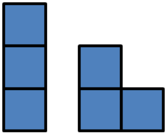


Figure: Trominoes

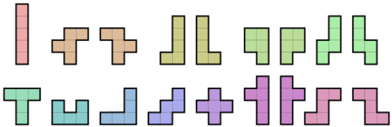
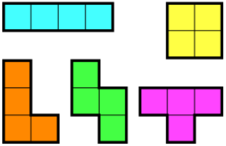


Figure: Pentominoes

Figure: Domino



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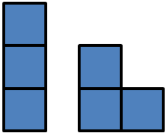
Classification of polyominoes

Figure: Monomino



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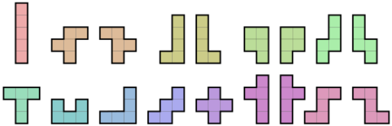

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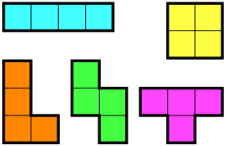
Figure: Domino



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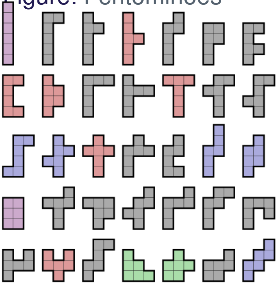

Figure: Hexominoes

Tilings with polyominoes. Homology group.

- Conway and Lagarias, Tiling with Polyominoes and Combinatorial group theory (1990)

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Definition (Homology group)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

Tilings with polyominoes. Homology group.

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- Michael Reid, Tile homotopy group (2003)
- Homology group

Definition (Homology group)

The tile homology group of Σ is the quotient $H(\Sigma) = A/B(\Sigma)$

- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ

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- where $B(\Sigma)$ is the subgroup generated by all elements corresponding to possible placements of tiles in Σ
- A is the free abelian group (on all the cells of the square lattice).

Homology groups of tilings

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- We consider whether exists a proper tiling of given region M (surface, surface with the boundary, etc.) subdivided into "cells" like grid with a tiles from a given set Σ.

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- A is free Abelian group on all the cells of given region M.

- A necessary condition for existence of a proper tiling is that the element corresponding to the sum of all cells of M is trivial in the homology group of tilings Σ.

Homology groups of generalized polyomino type tillings

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Example

Is it possible to tile torus chessboard 6×6 with tiles 1×4 (all orientation are allowed)?

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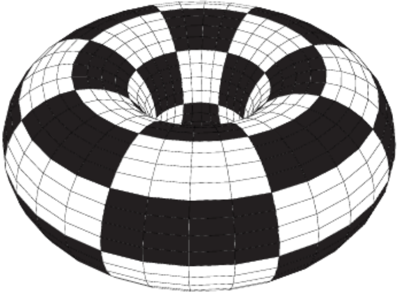

Figure: Torus Chessboard

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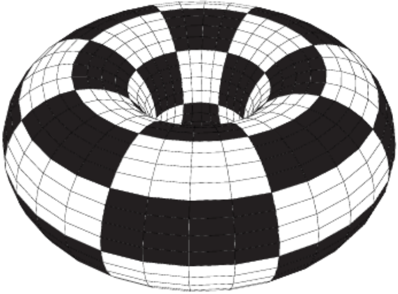


Figure: Torus Chessboard

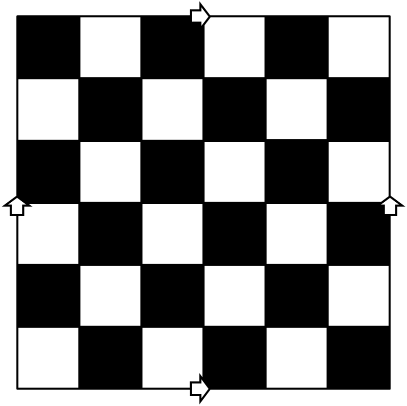


Figure: In torus plane model

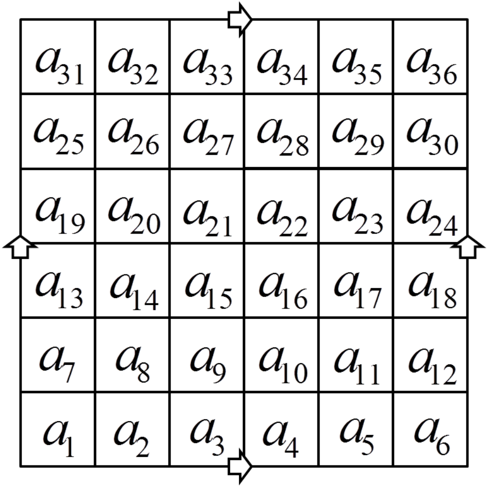


Figure: Naming cells

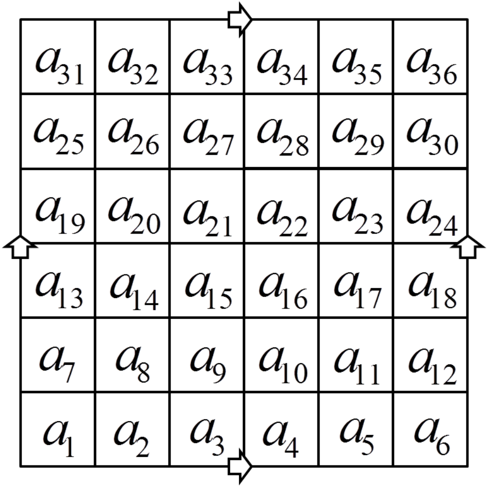


Figure: Naming cells

$$a_1 + a_2 + a_3 + a_4 = 0$$

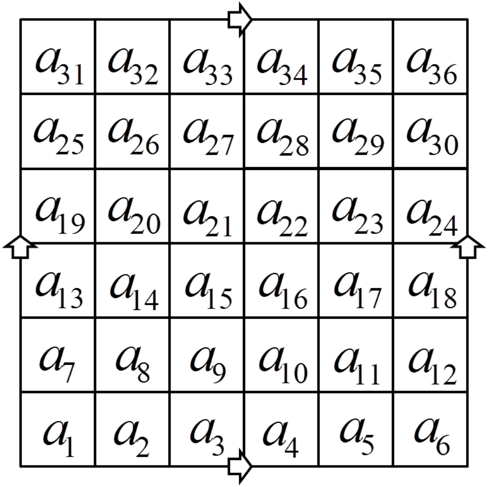
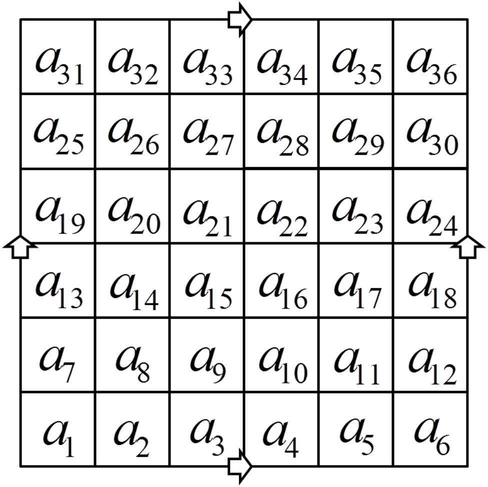


Figure: Naming cells

$$
\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0
\end{aligned}
$$



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$$a_1 + a_2 + a_3 + a_4 = 0$$
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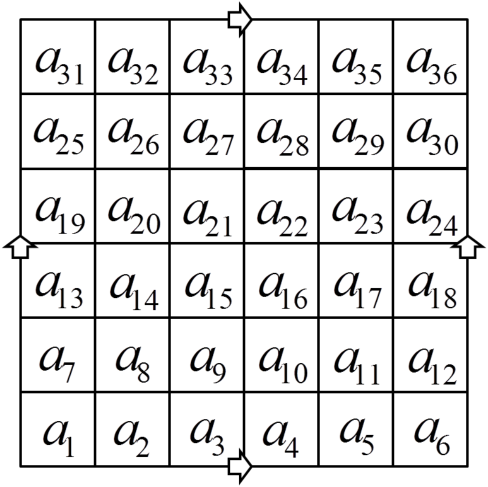
$$a_3 + a_4 + a_5 + a_6 = 0$$



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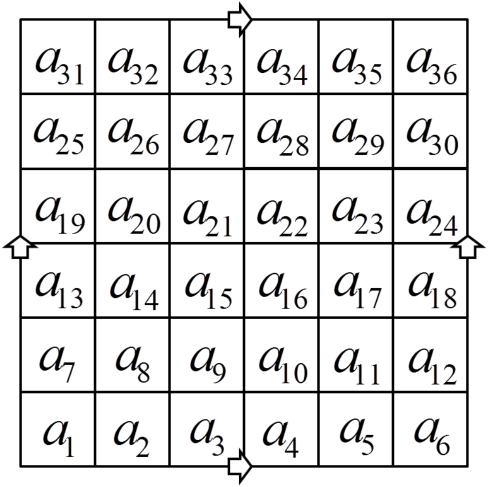
a_1 + a_2 + a_3 + a_4 &= 0 \\
a_2 + a_3 + a_4 + a_5 &= 0 \\
a_3 + a_4 + a_5 + a_6 &= 0 \\
a_4 + a_5 + a_6 + a_1 &= 0
\end{aligned}
$$



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a_4 + a_5 + a_6 + a_1 &= 0 \\
a_5 + a_6 + a_1 + a_2 &= 0
\end{aligned}
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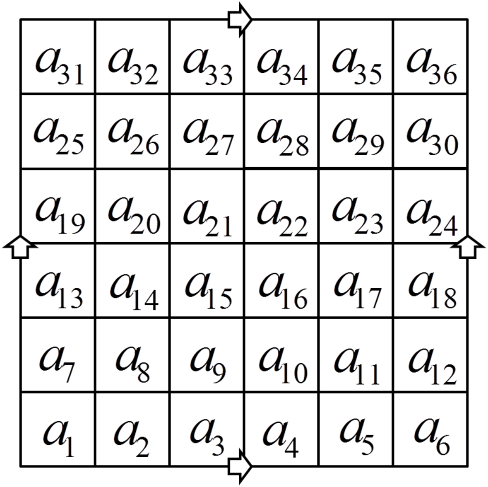

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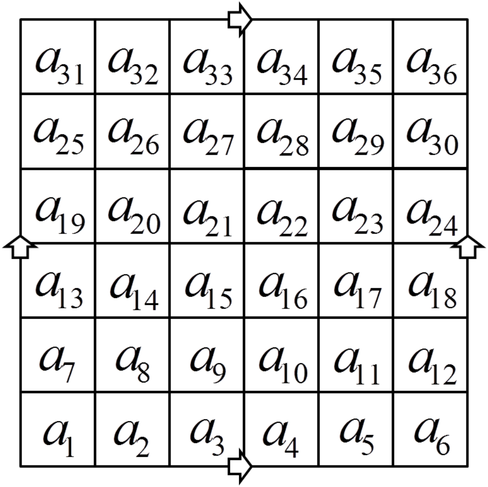
- relation in finite group



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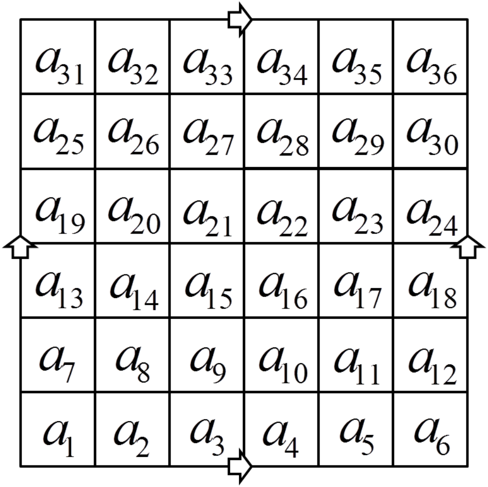

Figure: Naming cells

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\begin{aligned}
a_1 + a_2 + a_3 + a_4 &= 0 \\
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\end{aligned}
$$

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$$
\begin{aligned}
a_1 = a_5 = a_3 \\
a_2 = a_6 = a_4
\end{aligned}
$$

Analogue

$$a_7 = a_{11} = a_9$$
$$a_{13} = a_{17} = a_{15}$$
$$a_{19} = a_{23} = a_{21}$$
$$a_{25} = a_{29} = a_{27}$$
$$a_{31} = a_{35} = a_{33}$$
$$a_8 = a_{10} = a_{12}$$
$$a_{14} = a_{16} = a_{18}$$
$$a_{20} = a_{22} = a_{24}$$
$$a_{26} = a_{28} = a_{30}$$
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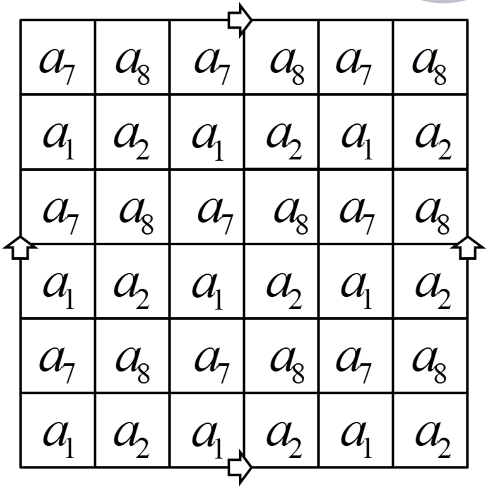
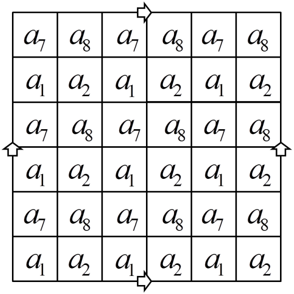


Figure: Equivalent cells



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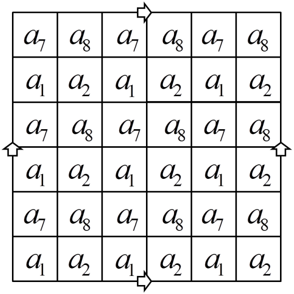
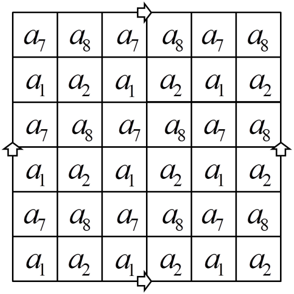


Figure: Equivalent cells

- if we put now tile 1×4 on our chessboard



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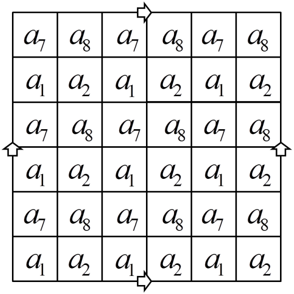
$$2a_1 + 2a_2 = 0$$



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$$2a_1 + 2a_2 = 0$$
$$2a_7 + 2a_8 = 0$$

a_7	a_8	a_7	a_8	a_7	a_8
a_1	a_2	a_1	a_2	a_1	a_2
a_7	a_8	a_7	a_8	a_7	a_8
a_1	a_2	a_1	a_2	a_1	a_2
a_7	a_8	a_7	a_8	a_7	a_8
a_1	a_2	a_1	a_2	a_1	a_2

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$$2a_1 + 2a_2 = 0 \qquad 2a_1 + 2a_7 = 0$$
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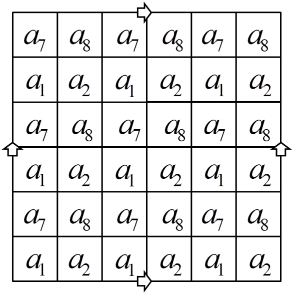

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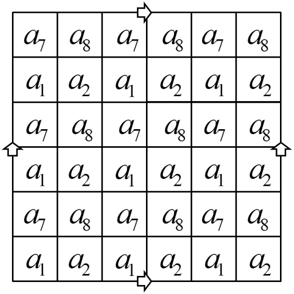

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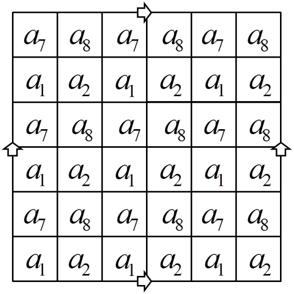
- 4 generators a_1, a_2, a_7, a_8



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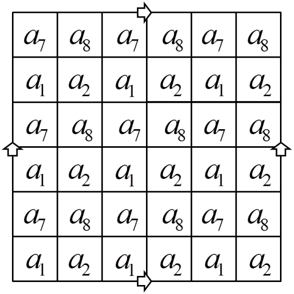

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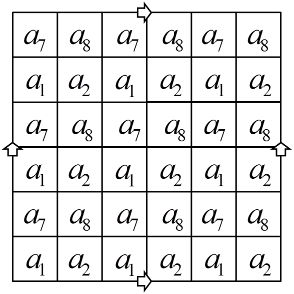
- 9 cells a_1,



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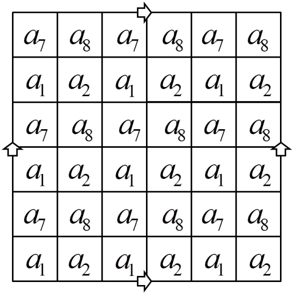

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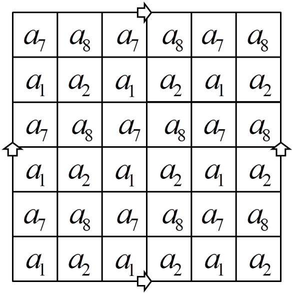

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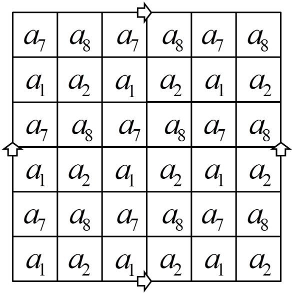

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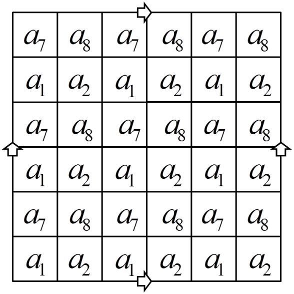
- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$



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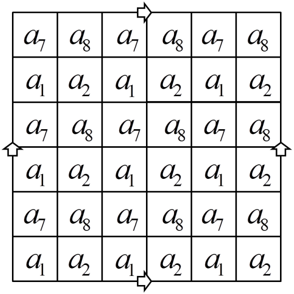
- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$

- non trivial element



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- 9 cells a_1, 9 cells a_2, 9 cells a_7, 9 cells a_8

$$a_1 + a_2 + a_7 + a_8$$

- non trivial element tiling is not possible

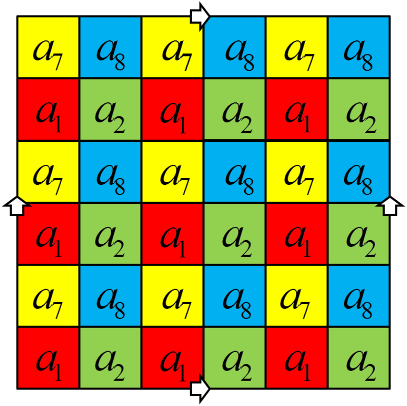


Figure: Coloring the Chessboard

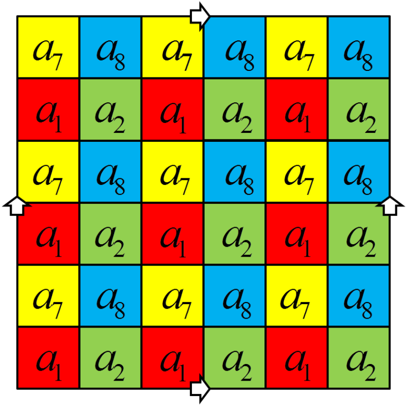


Figure: Coloring the Chessboard

- 9 red cells

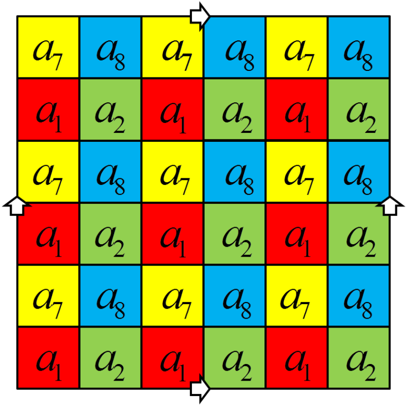


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells

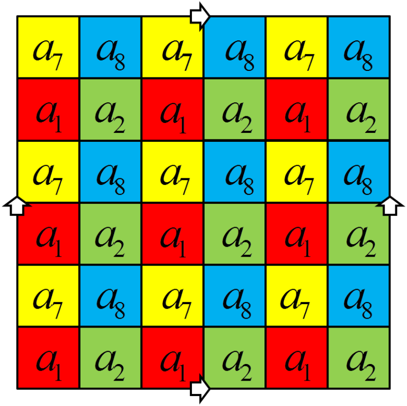


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells

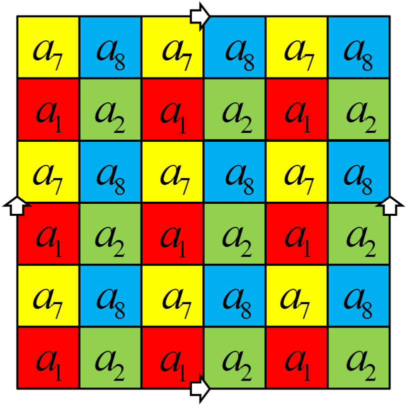


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells

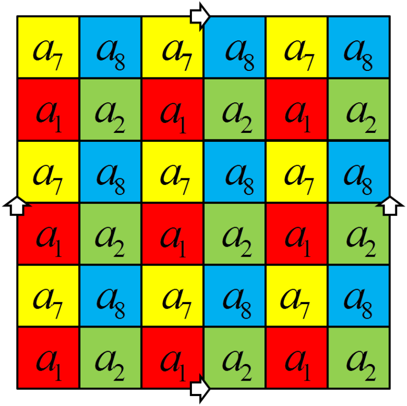


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering

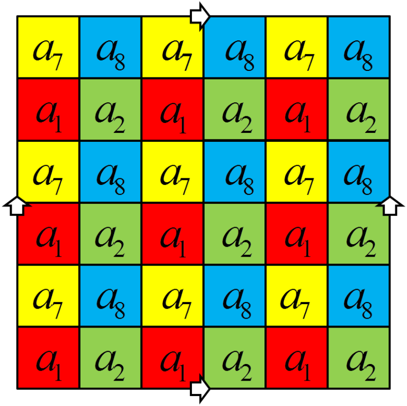
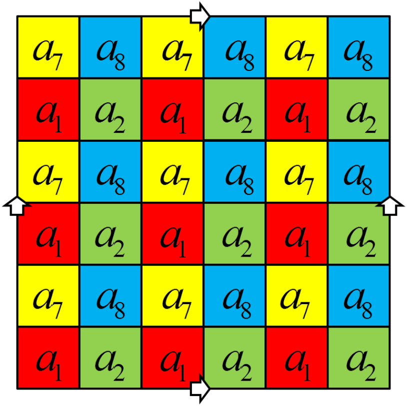


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green



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- 9 red cells
- 9 green cells
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- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue

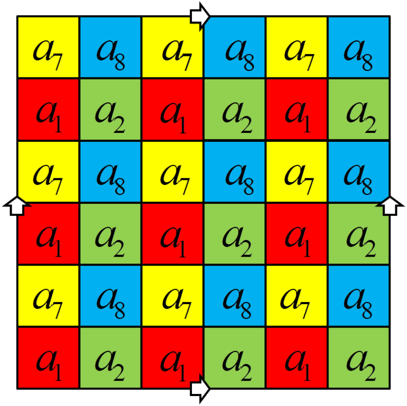
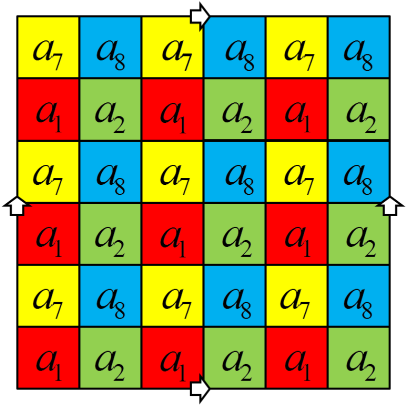


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow



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  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow
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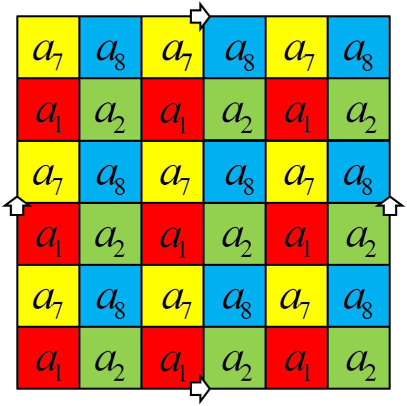
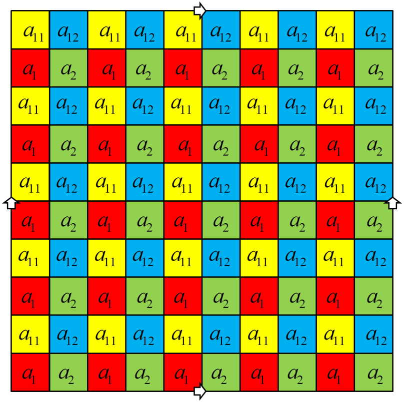


Figure: Coloring the Chessboard

- 9 red cells
- 9 green cells
- 9 yellow cells
- 9 blue cells
- every tile 1×4 covering
  - 2 red and 2 green
  - 2 yellow and 2 blue
  - 2 red and 2 yellow
  - 2 green and 2 blue
- tiling is not possible

□



Theorem

The torus chessboard of dimension $(4k+2) \times (4k+2)$ *can be not tiling with the tile* 1×4.

Example

Is it possible to tile torus chessboard 10×10 with T – tetrominoes? (all orientation are allowed)

Example

Is it possible to tile torus chessboard 10×10 with T – tetrominoes? (all orientation are allowed)

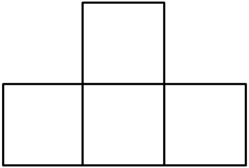


Figure: T – tetramino

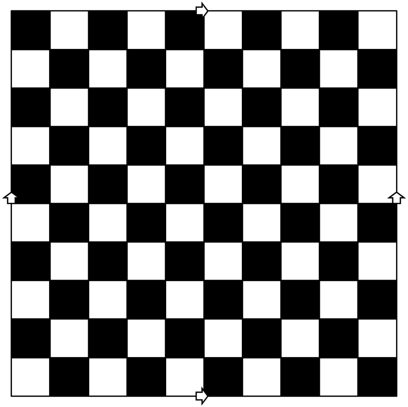


Figure: In torus plane model 10×10

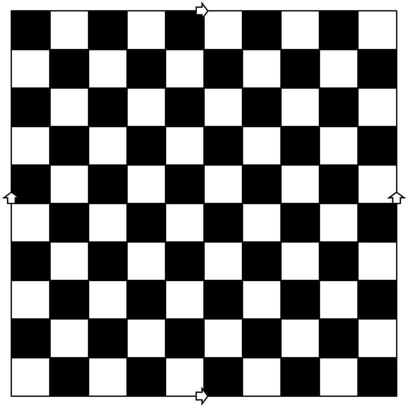


Figure: In torus plane model 10×10

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

Figure: Naming cells

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_1 + a_2 + a_3 + a_{12} = 0$$

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_1 + a_2 + a_3 + a_{12} = 0$$
$$a_2 + a_3 + a_4 + a_{13} = 0$$
$$a_3 + a_4 + a_5 + a_{14} = 0$$
$$a_4 + a_5 + a_6 + a_{15} = 0$$
$$a_5 + a_6 + a_7 + a_{16} = 0$$
$$a_6 + a_7 + a_8 + a_{17} = 0$$
$$a_7 + a_8 + a_9 + a_{18} = 0$$
$$a_8 + a_9 + a_{10} + a_{19} = 0$$
$$a_9 + a_{10} + a_1 + a_{20} = 0$$
$$a_{10} + a_1 + a_2 + a_{11} = 0$$

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

$$a_{11} + a_{12} + a_{13} + a_2 \quad = \quad 0$$

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

$$\begin{aligned} a_{11} + a_{12} + a_{13} + a_2 &= 0 \\ a_{11} + a_{12} + a_{13} + a_{22} &= 0 \end{aligned}$$

a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}
a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}
a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
a_{1}	a_{2}	a_{3}	a_{4}	a_{5}	a_{6}	a_{7}	a_{8}	a_{9}	a_{10}

$$a_{11} + a_{12} + a_{13} + a_2 = 0$$
$$a_{12} + a_{13} + a_{14} + a_3 = 0$$
$$a_{13} + a_{14} + a_{15} + a_4 = 0$$
$$a_{14} + a_{15} + a_{16} + a_5 = 0$$
$$a_{15} + a_{16} + a_{17} + a_6 = 0$$
$$a_{16} + a_{17} + a_{18} + a_7 = 0$$
$$a_{17} + a_{18} + a_{19} + a_8 = 0$$
$$a_{18} + a_{19} + a_{20} + a_9 = 0$$
$$a_{19} + a_{20} + a_{11} + a_{10} = 0$$
$$a_{20} + a_{11} + a_{12} + a_1 = 0$$

$$\begin{aligned} a_{11} + a_{12} + a_{13} + a_2 &= 0 \\ a_{11} + a_{12} + a_{13} + a_{22} &= 0 \end{aligned}$$

$$a_2 = a_{22}$$

$$a_1 = a_3 = a_5 = a_7 = a_9 = a_{12} = a_{14} = a_{16} = a_{18} = a_{20}$$
$$a_2 = a_6 = a_8 = a_{10} = a_{11} = a_{13} = a_{15} = a_{17} = a_{19}$$

$$a_1 = a_3 = a_5 = a_7 = a_9 = a_{12} = a_{14} = a_{16} = a_{18} = a_{20}$$
$$a_2 = a_6 = a_8 = a_{10} = a_{11} = a_{13} = a_{15} = a_{17} = a_{19}$$

Analogue

$$a_{21} = a_{23} = a_{25} = a_{27} = a_{29} = a_{32} = a_{34} = a_{36} = a_{38} = a_{40}$$
$$a_{41} = a_{43} = a_{45} = a_{47} = a_{49} = a_{42} = a_{44} = a_{46} = a_{48} = a_{60}$$
$$a_{61} = a_{63} = a_{65} = a_{67} = a_{69} = a_{62} = a_{64} = a_{66} = a_{68} = a_{80}$$
$$a_{81} = a_{83} = a_{85} = a_{87} = a_{89} = a_{82} = a_{84} = a_{86} = a_{88} = a_{100}$$
$$a_{22} = a_{24} = a_{26} = a_{28} = a_{30} = a_{31} = a_{33} = a_{35} = a_{37} = a_{39}$$
$$a_{42} = a_{44} = a_{46} = a_{48} = a_{50} = a_{51} = a_{53} = a_{55} = a_{57} = a_{59}$$
$$a_{62} = a_{64} = a_{66} = a_{68} = a_{70} = a_{71} = a_{73} = a_{75} = a_{77} = a_{79}$$
$$a_{82} = a_{84} = a_{86} = a_{88} = a_{90} = a_{91} = a_{93} = a_{95} = a_{97} = a_{99}$$

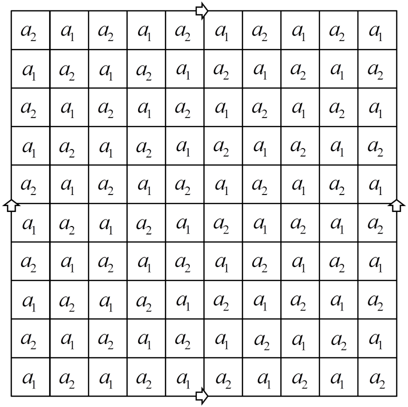


Figure: Equivalent cells

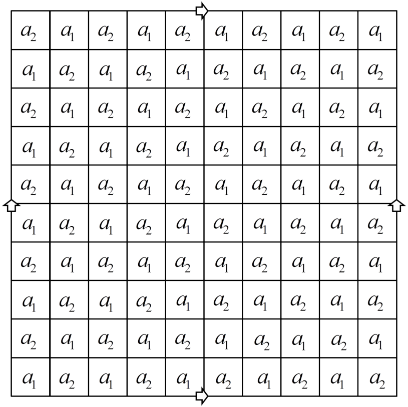


Figure: Equivalent cells

$$3a_1 + a_2 = 0$$

a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2

Figure: Equivalent cells

$$
\begin{aligned}
3a_1 + a_2 &= 0 \\
3a_2 + a_1 &= 0
\end{aligned}
$$

a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1
a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2

Figure: Equivalent cells

$$\begin{aligned} 3a_1 + a_2 &= 0 \\ 3a_2 + a_1 &= 0 \end{aligned}$$

- $< a_1, a_2 |_{3a_1+a_2, 3a_2+a_1} >=< a_1 |_{8a_1=0} >= \mathbb{Z}_8$

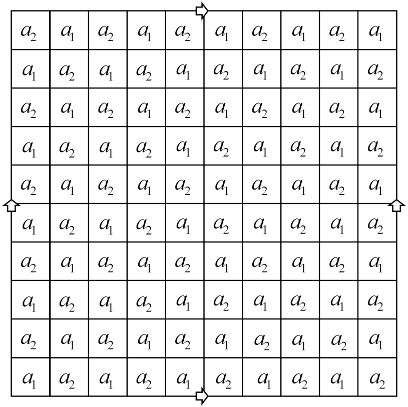


Figure: Equivalent cells

$$
\begin{aligned}
3a_1 + a_2 &= 0 \\
3a_2 + a_1 &= 0
\end{aligned}
$$

- $< a_1, a_2|_{3a_1+a_2, 3a_2+a_1} >=< a_1|_{8a_1=0} >= \mathbb{Z}_8$

$$50a_1 + 50a_2 = -100a_1 = 4a_1$$

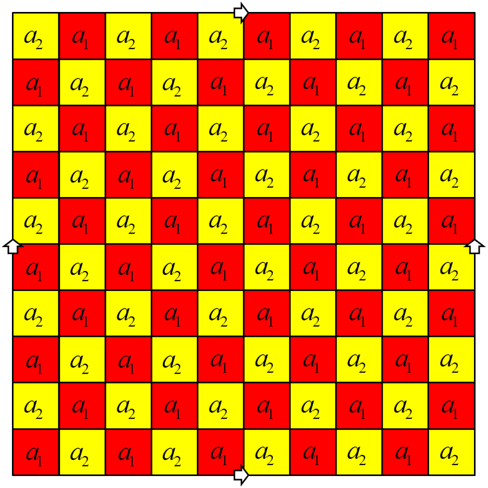


Figure: Coloring the Chessboard

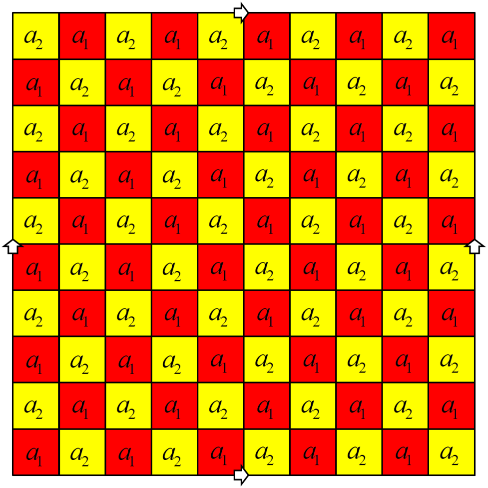


Figure: Coloring the Chessboard

- 50 red cells

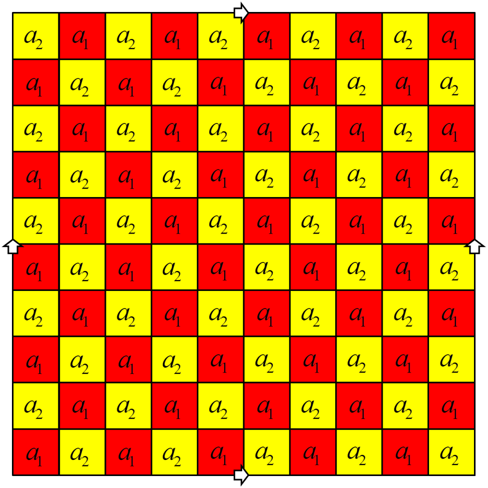


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells

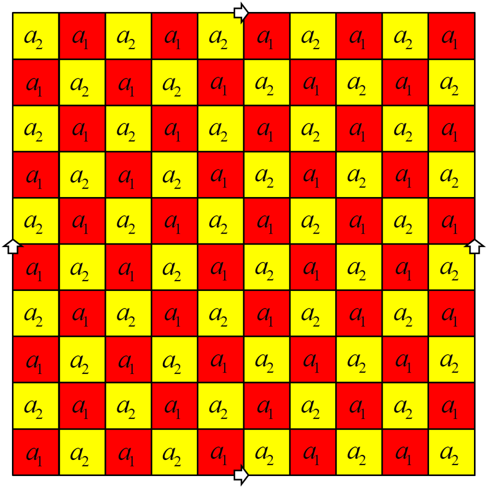


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover

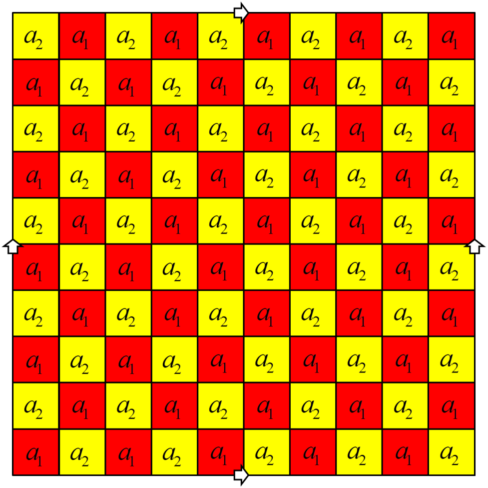


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow

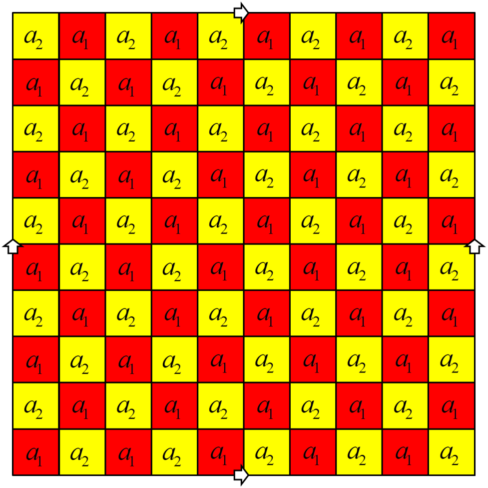


Figure: Coloring the Chessboard

- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow
  - 3 yellow and 1 red

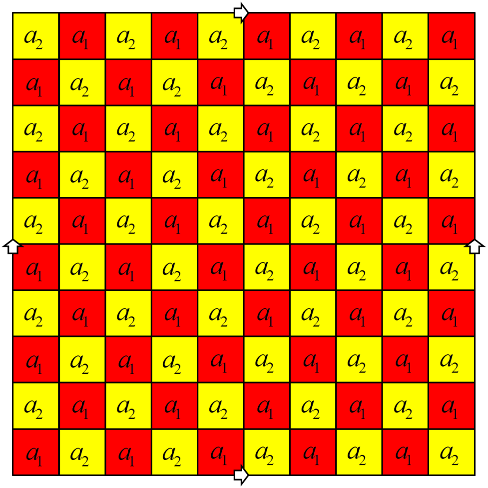


Figure: Coloring the Chessboard

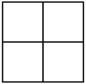
- 50 red cells
- 50 yellow cells
- every T – tetramino cover
  - 3 red and 1 yellow
  - 3 yellow and 1 red
- tiling is not possible

Example

Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?

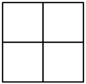
Example

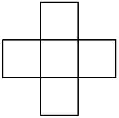
Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?



Example

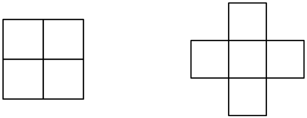
Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?





Example

Is it possible to tile torus chessboard 9×5 with one removed cell in the middle to tile with square shapes 2×2 and cross shape (all orientation are allowed)?



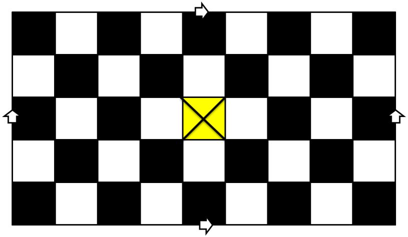


Figure: In torus plane model

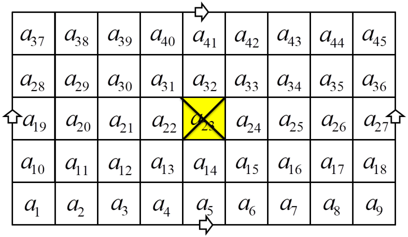
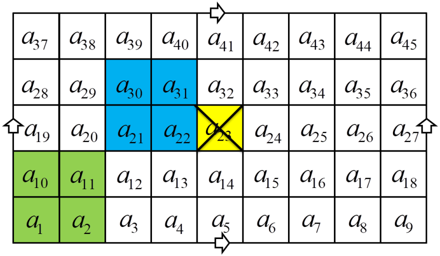
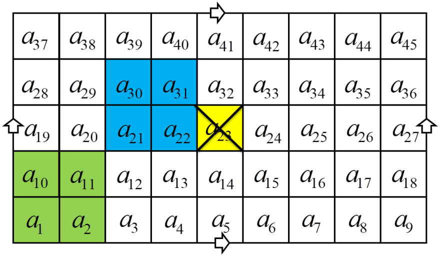


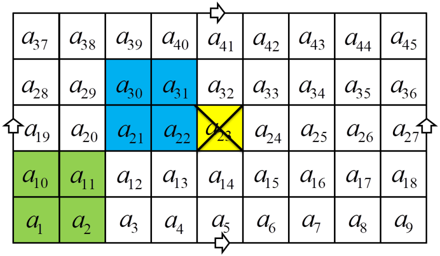
Figure: Naming cell



a37 a38 a39 a40 a41 a42 a43 a44 a45
a28 a29 a30 a31 a32 a33 a34 a35 a36
a19 a20 a21 a22 a23 a24 a25 a26 a27
a10 a11 a12 a13 a14 a15 a16 a17 a18
a1 a2 a3 a4 a5 a6 a7 a8 a9

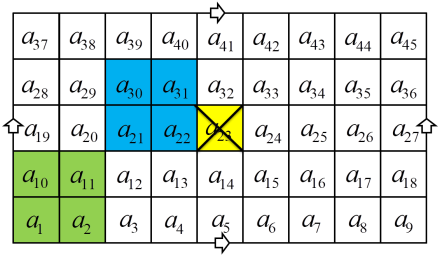


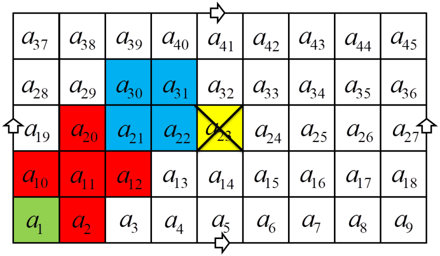
$$a_1 + a_2 + a_{10} + a_{11} = 0$$



$$a_1 + a_2 + a_{10} + a_{11} = 0$$

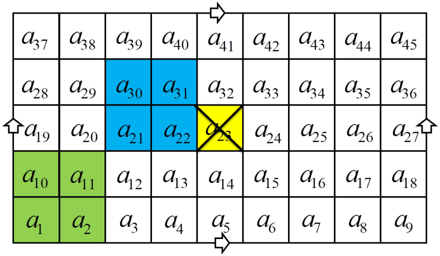
$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$

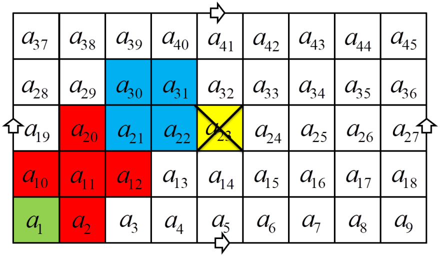




$$a_1 + a_2 + a_{10} + a_{11} = 0$$

$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$

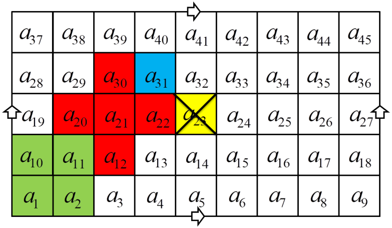




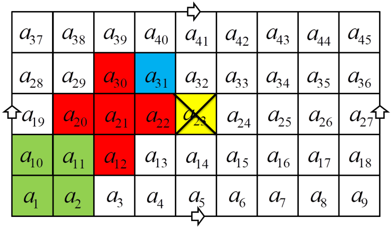
$$a_1 + a_2 + a_{10} + a_{11} = 0$$

$$a_{21} + a_{30} + a_{22} + a_{31} = 0$$

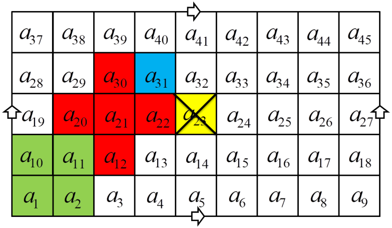
$$a_{20} + a_{10} + a_{11} + a_{12} + a_2 = 0$$



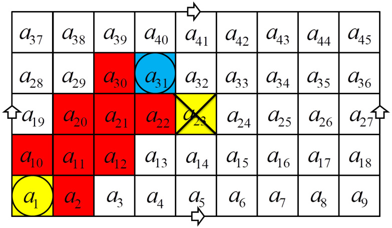
a37 a38 a39 a40 a41 a42 a43 a44 a45
a28 a29 a30 a31 a32 a33 a34 a35 a36
a19 a20 a21 a22 a23 a24 a25 a26 a27
a10 a11 a12 a13 a14 a15 a16 a17 a18
a1 a2 a3 a4 a5 a6 a7 a8 a9

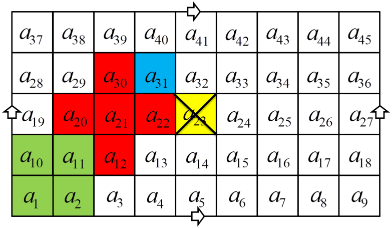


$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$



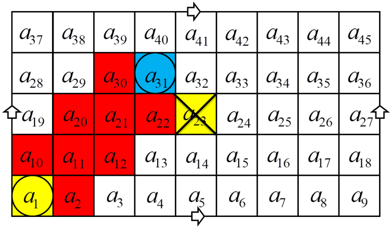
$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$

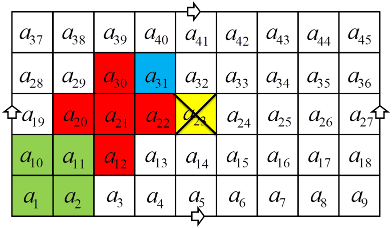




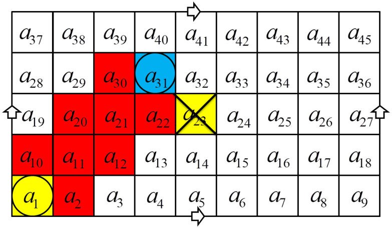
$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$

$$a_1 = a_{31}$$





$$a_{12} + a_{20} + a_{21} + a_{22} + a_{30} = 0$$



$$a_1 = a_{31}$$

Analogue

$$a_{31} = a_{16} = a_{37} = a_{22}$$
$$a_1 = a_{34} = a_{10} = a_{40} = a_{25}$$
$$a_{37} = a_{13} = a_{28}$$
$$a_{28} = a_4 = a_{19} = a_{43}$$
$$a_{28} = a_7$$

a_1	a_{38}	a_{39}	a_1	a_{41}	a_{42}	a_1	a_{44}	a_{45}
a_1	a_{29}	a_{30}	a_1	a_{32}	a_{33}	a_1	a_{35}	a_{36}
a_1	a_{20}	a_{21}	a_1	X	a_{24}	a_1	a_{26}	a_{27}
a_1	a_{11}	a_{12}	a_1	a_{14}	a_{15}	a_1	a_{17}	a_{18}
a_1	a_2	a_3	a_1	a_5	a_6	a_1	a_8	a_9

Figure: Cells generated with a_1

a_1	a_{38}	a_{39}	a_1	a_{41}	a_{42}	a_1	a_{44}	a_{45}
a_1	a_{29}	a_{30}	a_1	a_{32}	a_{33}	a_1	a_{35}	a_{36}
a_1	a_{20}	a_{21}	a_1	X	a_{24}	a_1	a_{26}	a_{27}
a_1	a_{11}	a_{12}	a_1	a_{14}	a_{15}	a_1	a_{17}	a_{18}
a_1	a_2	a_3	a_1	a_5	a_6	a_1	a_8	a_9

Figure: Cells generated with a_1

Analogue

$$a_2 = a_{34} = a_5, a_{34} = a_{11} = a_{41} = a_{26}$$

$$a_{11} = a_{44}, a_{20} = a_5, a_{29} = a_{14} = a_{34}$$

$$a_8 = a_{29}, a_{38} = a_{17} = a_{32}, a_{17} = a_{29}$$

a_1	a_{38}	a_{39}	a_1	a_{41}	a_{42}	a_1	a_{44}	a_{45}
a_1	a_{29}	a_{30}	a_1	a_{32}	a_{33}	a_1	a_{35}	a_{36}
a_1	a_{20}	a_{21}	a_1	X	a_{24}	a_1	a_{26}	a_{27}
a_1	a_{11}	a_{12}	a_1	a_{14}	a_{15}	a_1	a_{17}	a_{18}
a_1	a_2	a_3	a_1	a_5	a_6	a_1	a_8	a_9

Figure: Cells generated with a_1

a_1	a_2	a_{39}	a_1	a_{41}	a_2	a_1	a_2	a_{45}
a_1	a_2	a_{30}	a_1	a_{32}	a_2	a_1	a_2	a_{36}
a_1	a_2	a_{21}	a_1	X	a_2	a_1	a_2	a_{27}
a_1	a_2	a_{12}	a_1	a_{14}	a_2	a_1	a_2	a_{18}
a_1	a_2	a_3	a_1	a_5	a_2	a_1	a_2	a_9

Figure: Cells generated with a_2

Analogue

$$a_2 = a_{34} = a_5, a_{34} = a_{11} = a_{41} = a_{26}$$
$$a_{11} = a_{44}, a_{20} = a_5, a_{29} = a_{14} = a_{34}$$
$$a_8 = a_{29}, a_{38} = a_{17} = a_{32}, a_{17} = a_{29}$$

a_1	a_{38}	a_{39}	a_1	a_{41}	a_{42}	a_1	a_{44}	a_{45}
a_1	a_{29}	a_{30}	a_1	a_{32}	a_{33}	a_1	a_{35}	a_{36}
a_1	a_{20}	a_{21}	a_1	X	a_{24}	a_1	a_{26}	a_{27}
a_1	a_{11}	a_{12}	a_1	a_{14}	a_{15}	a_1	a_{17}	a_{18}
a_1	a_2	a_3	a_1	a_5	a_6	a_1	a_8	a_9

Figure: Cells generated with a_1

a_1	a_2	a_{39}	a_1	a_{41}	a_2	a_1	a_2	a_{45}
a_1	a_2	a_{30}	a_1	a_{32}	a_2	a_1	a_2	a_{36}
a_1	a_2	a_{21}	a_1	X	a_2	a_1	a_2	a_{27}
a_1	a_2	a_{12}	a_1	a_{14}	a_2	a_1	a_2	a_{18}
a_1	a_2	a_3	a_1	a_5	a_2	a_1	a_2	a_9

Figure: Cells generated with a_2

Analogue

$$a_2 = a_{34} = a_5, a_{34} = a_{11} = a_{41} = a_{26}$$
$$a_{11} = a_{44}, a_{20} = a_5, a_{29} = a_{14} = a_{34}$$
$$a_8 = a_{29}, a_{38} = a_{17} = a_{32}, a_{17} = a_{29}$$

Analogue

$$a_3 = a_{36} = a_6 = a_{27}$$
$$a_{39} = a_{18} = a_{42} = a_{27}, a_{12} = a_{33}$$
$$a_{33} = a_9, a_{21}, a_{24} = a_9, a_{30} = a_{15}$$

a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	X	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3

Figure: Cells generated with a_3

a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	X	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3

Figure: Cells generated with a_3

$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$

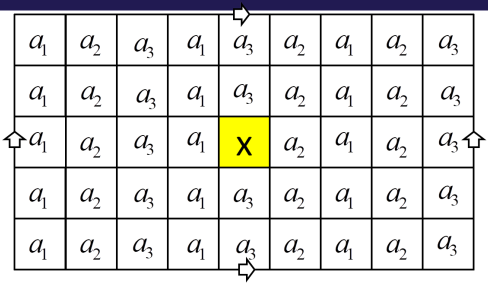


Figure: Cells generated with a_3

$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$

a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	X	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3

Figure: Cells generated with a_3

$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$

- $< a_1, a_2, a_3 | 2a_1 = 2a_2 = 2a_3 = a_1 + a_2 + a_3 = 0 >$

a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	X	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3
a_1	a_2	a_3	a_1	a_3	a_2	a_1	a_2	a_3

Figure: Cells generated with a_3

$$2a_1 + 2a_2 = 0$$
$$2a_2 + 2a_3 = 0$$
$$2a_1 + 2a_3 = 0$$
$$3a_2 + a_1 + a_3 = 0$$
$$3a_3 + a_1 + a_2 = 0$$
$$3a_1 + a_2 + a_3 = 0$$

- $< a_1, a_2, a_3 | 2a_1 = 2a_2 = 2a_3 = a_1 + a_2 + a_3 = 0 >$

$$15a_1 + 14a_2 + 15a_3 = a_1 + a_3$$

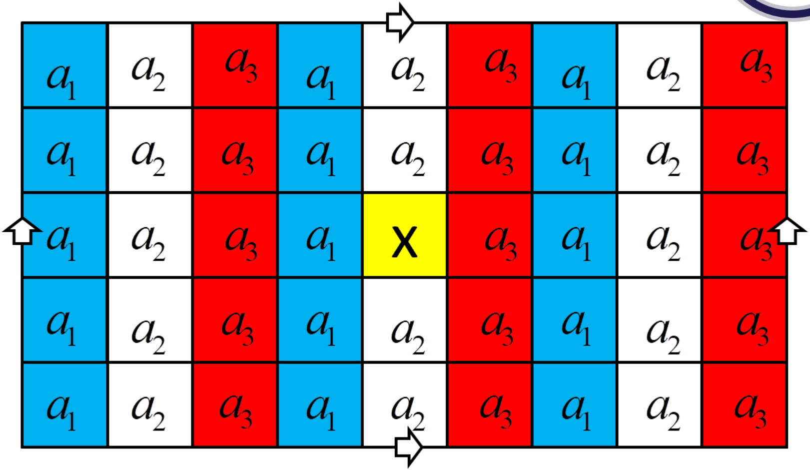
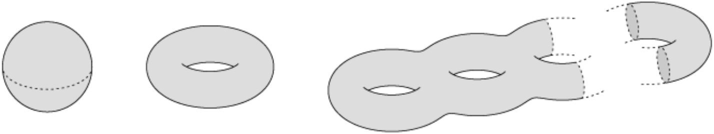


Figure: Coloring the chessboard

Conclusion

- The same idea can be used for studying tilings on surfaces of genus g. Which are subdivided in more general cells grids.



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